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MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD

Conferencias plenarias / *Plenary talks*

Counting modular forms with a Galois representation mod p and the Atkin-Lehner eigenvalue at p fixed simultaneously

Samuele Anni[1]

The structure of the algebra of modular forms over finite fields has been widely studied, in part for its applications in establishing congruences. In this talk, after recalling classical geometric arguments of Ogg and Kenku, I will show how, for N prime with p, one can count the number of classical modular forms of level Np and weight k with both a residual Galois representation and an Atkin-Lehner sign at fixed p, generalizing Martin's recent results, and dimension formulas given by Jochnowitz and by Bergdall-Pollack.

Most of these results can be stated as equivariant isomorphisms for the Hecke operators between certain modules, thanks to a p-adic refinement of the Brauer-Nesbitt theorem. A theoretical framework for proving such isomorphisms is given, using the Eichler-Selberg trace formula. This method applies in the case where the level is divisible by the residual characteristic, contrary to the pre-existing approaches.

This is work in progress with Alexandru Ghitza (University of Melbourne) and Anna Medvedovsky (Boston University)

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On Shimura curves of PEL-type

Pilar Bayer[1]

Shimura curves of PEL-type are a remarkable generalization of modular curves. We shall consider those defined by means of non-split rational quaternion algebras and will give their interpretation as coarse moduli spaces for fake elliptic curves. A convenient generalization of the classical complex multiplication theory, led G. Shimura to his theory of the canonical models. As in the modular case, fake elliptic curves with complex multiplication play a key role in the theoretical construction of class fields by means of special values of arithmetic automorphic functions.

In joint work with M. Alsina and J. Guàrdia, I shall present a method to obtain the equations of the fake elliptic curves defined by special complex multiplication points in the canonical model.

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On a generalization of Fourier analysis arising from number theory

Pablo Candela[1]

In analytic and combinatorial number theory, many applications of Fourier analysis rely on the following idea: the averages of a function over certain linear patterns in an abelian group can be usefully analyzed by approximating the function using its dominating Fourier components (this is the case in particular in the circle method).

In the last two decades, this idea has been considerably extended by the progress of a theory known as higher-order Fourier analysis. This theory grew out of seminal work of Gowers concerning a central result in combinatorial number theory, namely Szemerédi's theorem. The theory has also powered celebrated results such as asymptotic estimates for counts of various types of linear patterns in the primes, notably in work of Green and Tao. A key insight in higher-order Fourier analysis is that for many types of linear configurations, while the approximations by dominating Fourier components may not be helpful, it is still possible to carry out a very useful analysis by approximating the function by dominating components that are not defined using the circle group anymore, but rather using certain inherently non-commutative generalizations, such as nilmanifolds. Among the developments generated by this insight, there is the study of fascinating structures called nilspaces. These structures constitute a common generalization of abelian groups and nilmanifolds, and have yielded further progress in the theory of higher-order Fourier analysis. I will provide an introduction to this theory and discuss some recent results in this approach involving nilspaces.

The talk is based on joint work with Balázs Szegedy.

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Sobre el grupo de componentes del grupo de Sato-Tate

Victoria Cantoral-Farfán[1]

La famosa conjetura de Sato-Tate para curvas elípticas (sin multiplicación compleja y definidas sobre un campo de números) predice la equidistribución de las trazas de los automorfismos de Frobenius con respecto a la medida de Haar del correspondiente grupo de Sato-Tate. Esta conjetura ya se ha generalizada para variedades abelianas de mayor dimensión, superficies $K3$ y motivos puros de peso impar. Es natural estudiar con más detalle el grupo Sato-Tate para poder en un futuro abordar la conjetura generalizada de Sato-Tate.

Durante esta charla presentaremos esta conjetura y describiremos en detalle el grupo de componentes del grupo Sato-Tate de una variedad abeliana de dimensión arbitraria, definida sobre un campo de números K.

Este es un trabajo conjunto con Grzegorz Banaszak.

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Rigid meromorphic cocycles

Henri Darmon[1]

This talk will describe the objects in the title and explain their relevance to explicit class field theory and to extending the theory of Borcherds' singular theta lifts to orthogonal groups of general real signature.

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Computing plectic Stark-Heegner invariants

Marc Masdeu[1]

Let E/F be an elliptic curve defined over a number field F, and let K/F be a quadratic extension. If the analytic rank of $E(K)$ is one, one can often use Heegner points (or the more general Star-Heegner points) to produce (at least conjecturally) a nontorsion generator of $E(K)$. If the analytic rank of $E(K)$ is larger than one, the problem of constructing algebraic points is still very open. In recent work, Michele Fornea and Lennart Gehrmann have introduced certain p-adic quantities that may be conjecturally related to the existence of these points. In this talk I will explain their construction, and illustrate with numerical experiments some support for their conjecture.

This is joint work with Michele Fornea and Xevi Guitart.

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Sumar $\mu(n)$: un algoritmo elemental más rápido

Harald Andrés Helfgott[1], **Lola Thompson**[2]

Presentamos un nuevo algoritmo elemental para calcular

$$M(x) = \sum_{n\leq x} \mu(n),$$

donde $\mu(n)$ es la función de Möbius. Nuestro algoritmo toma tiempo $O_\varepsilon\left(x^{\frac{3}{5}}(\log x)^{\frac{3}{5}+\varepsilon}\right)$ y espacio $O\left(x^{\frac{3}{10}}(\log x)^{\frac{13}{10}}\right)$, lo cual mejora los algoritmos combinatorios existentes. Si bien existe un algoritmo analítico de Lagarias-Odlyzko con cálculos basados en integrales de $\zeta(s)$ que solo toma tiempo $O(x^{1/2+\epsilon})$, nuestro algoritmo tiene la ventaja de ser más fácil de implementar. El nuevo enfoque equivale aproximadamente a analizar la diferencia entre un modelo que obtenemos a través de la aproximación diofántica y la realidad, y a mostrar que tiene una descripción simple en términos de clases de congruencia y segmentos. Esta simple descripción nos permite calcular la diferencia rápidamente por medio de búsquedas en tablas.

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Comunicaciones cortas / *Short talks*

Sistemas de Euler anticiclotómicos y ciclos diagonales II

Raúl Alonso[1], **Francesc Castellà**[2], **Óscar Rivero**[3]

En la presentación a cargo de Óscar Rivero (titulada «Sistemas de Euler anticiclotómicos y ciclos diagonales I») se discutieron algunos avances recientes en la teoría de sistemas de Euler, así como la motivación detrás de los trabajos [1] y [2].

En esta presentación explicaremos con más detalle cómo obtener un sistema de Euler para la convolución de dos formas modulares a partir de familias p-ádicas de ciclos diagonales. Esta construcción constituye una parte esencial del trabajo [1] y esperamos que algunos aspectos de la misma se puedan extender a otras situaciones. *Grosso modo*, la idea consiste en especializar la construcción de familias p-ádicas de ciclos diagonales llevada a cabo en los trabajos [3] y [4], asociada al producto de tres familias de Hida, al caso en que una de estas familias de Hida es una familia CM. De esta manera, esta familia de Hida se convierte en una variable anticiclotómica que nos permite obtener la variación deseada en la dirección p-ramificada. Variando la parte del nivel coprima con p en la construcción y explotando la geometría de los ciclos junto con resultados del trabajo [5], conseguimos obtener un conjunto de clases Λ-ádicas que satisfacen las relaciones de compatibilidad requeridas para constituir un sistema de Euler en este contexto.

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On infinity of cubic points over the rationals for quotient modular curves

Francesc Bars[1], **Tarun Dalal**[2]

A non-singular smooth curve C over a number field K of genus g_C has always a finite set of K-rational points $C(K)$ by a celebrated result of Faltings (here we fix once and for all $\overline{K}$, an algebraic closure of K). We consider the set of all points of degree at most d for C by $\Gamma_d(C,K) = \cup_{[L:K]\leq d} C(L)$ and exact degree d by $\Gamma'_d(C,K) = \cup_{[L:K]=d} C(L)$, where $L \subseteq \overline{K}$ runs over the finite extensions of K. A point $P \in C$ is said to be a point of degree d over K if $[K(P):K] = d$.

The set $\Gamma_d(C,M)$ is infinite for a certain M/K finite extension if C admits a degree at most d map, all defined over M, to a projective line or an elliptic curve with positive M-rank. The converse is true for $d=2$ [4], $d=3$ [1] and $d=4$ under certain restrictions [1, 3].

For $d=3$, Daeyeol Jeon introduced an arithmetic statement (i.e. fixing the field K in $\Gamma'_d(C,K)$) and its proof in [5] following [1] and [3]. In particular if $g_C \geq 3$ and C has no degree 3 or 2 map to a projective line and no degree 2 map to an elliptic curve over $\overline{K}$ then the set of exact cubic points of C over K, $\Gamma'_3(C,K)$, is an infinite set if and only if C admits a degree three map to an elliptic curve over K with positive K-rank.

The aim of the talk is to introduce the above Jeon results and different techniques and ideas that are used to determine when $\Gamma'_3(X_0^+(N),\mathbb{Q})$ (and for different quotient modular curves but not $X_0^*(N)$) is a finite or infinite set quoted in the preprint [2].

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La conjetura de Erdős-Straus

Manuel Bello-Hernández[1], **Manuel Benito**[2], **Emilio Fernández**[1]

En esta charla presentaremos el estado actual de la conjetura de Erdős-Straus. Paul Erdős y Ernest Straus conjeturaron a finales de la década de 1940 que, *dado un número natural* $n \geq 2$, *siempre existen números naturales* x, y, z *que cumplen*

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}.$$

Está conjetura combina las propiedades aditivas y multiplicativas de los números naturales y sigue abierta en la actualidad. Comentaremos además varias conjeturas relacionadas con ella. Algunas de las referencias donde se puede encontrar información sobre este tema son [1], [2] y [3].

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The modular method and its limitations

Pedro-José Cazorla García[1]

After Andrew Wiles's celebrated proof of Fermat's Last Theorem [2], a new toolkit of techniques was developed in order to completely solve certain Diophantine equations. These methods, based mainly on the Modularity Theorem proved by Wiles and on Ribet's Level Lowering Theorem [1], are now collectively known as the modular method.

In the last two decades, the modular method has been successfully applied to a plethora of Diophantine equations. However, there are theoretical and computational limitations to the method which were not apparent during the proof of Fermat's last theorem.

In this talk, we will briefly explain how to use the modular method and present the difficulties which may arise during its application. We will do this by showing examples which are not solvable by using only the modular methodology.

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On Drinfeld oldforms and newforms

Tarun Dalal[1]

The theory of oldforms and newforms is a well-understood area in the theory of classical modular forms. Certain properties of modular forms heavily depend on whether they belong to oldforms or newforms. For example, the space of newforms has a basis consisting of normalized eigenforms for all the Hecke operators. In fact, the Fourier coefficients of these normalized eigenforms generate a number field. However, the analogues theory of oldforms and newforms is not known for Drinfeld modular forms.

Recently, in a series of articles, Bandini and Valentino defined the notion of $\mathfrak{p}$-oldforms and $\mathfrak{p}$-newforms for Drinfeld modular forms. Moreover, they conjectured that the space of cusp for level T can be written as a direct sum decomposition of T-oldforms and T-newforms.

In this talk, we shall prove that, under certain assumptions, this conjecture is true. Moreover, we shall state a variant of this conjecture for higher level and provide many examples where this conjecture is true. However, for Drinfeld cusp forms of composite level, we shall prove that this conjecture may fail by providing examples of such spaces. Finally, we define the notion of oldforms and newforms for Drinfeld modular form of square free level. This is a joint work with Narasimha Kumar.

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Formalizing the ring of adèles and some applications in Lean

María Inés de Frutos Fernández[1]

We will present a formalization of the ring of adèles and group of idèles of a global field in Lean 3 [1]. Lean is an interactive theorem prover with an ever-growing mathematics library. We will give a quick introduction to Lean and explain how these definitions were formalized, with a focus on the kind of decisions one has to make during the formalization process.

As a prerequisite, we formalized adic valuations on Dedekind domains and their fields of fractions. After discussing the formalization of the adèle ring, we will present some applications, including the statement of the main theorem of global class field theory and a proof that the ideal class group of a number field is isomorphic to an explicit quotient of its idèle class group.

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Abelian varieties of GL_4 type

Enric Florit[1]

We say an abelian variety A is of GL_4-type if its endomorphism algebra contains a number field of degree $\frac{1}{2}\dim A$. Two kinds of GL_4-type abelian varieties have been included in the Paramodular Conjecture: abelian surfaces with trivial endomorphism ring and fourfolds with QM defined over $\mathbb{Q}$.

In this talk we present an initial study of these varieties. We define the concept of a GL_4-type building block, and we classify their possible endomorphism algebras. Then we study the ℓ-adic and λ-adic representations, and we relate them with inner twists whenever the Rosati involution is of the first kind. Finally, we introduce families of $\mathbb{Q}$-surfaces defined over certain quadratic fields, building on previous work of Hasegawa and using Richelot correspondences. This is joint work with Francesc Fité and Xevi Guitart.

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Towards a refined class number formula for Drinfeld modules

María Inés de Frutos Fernández[1], **Daniel Macías Castillo**[2], **Daniel Martínez Marqués**[2]

In 2012, Taelman proved an analogue of the Analytic Class Number Formula, for the Goss L-functions that are associated to Drinfeld modules. He also explicitly stated that 'it should be possible to formulate and prove an equivariant version' of this formula.

We report on the preliminary stages of work in progress, motivated by Taelman's statement. We formulate a precise equivariant, or 'refined', generalisation of Taelman's formula, and we provide very strong evidence to support it.

As a concrete consequence of our general approach, we also prove (unconditionally) a natural non-abelian generalisation of a theorem of Anglès and Taelman.

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On Dunkl zeta functions and their properties

Alejandro Gil Asensi[1], **Juan L. Varona**[1]

Dunkl theory [3] involves some tools such as the Dunkl derivative

$$\Lambda_\alpha f(x) = \frac{d}{dx} f(x) + \frac{2\alpha+1}{2}\frac{f(x)-f(-x)}{x}$$

or the Dunkl exponential

$$E_\alpha(z) = \mathcal{I}_\alpha(z) + \frac{z}{2(\alpha+1)}\mathcal{I}_{\alpha+1}(z),$$

where $\mathcal{I}_\alpha(z) = 2^\alpha\Gamma(\alpha+1)\frac{J_\alpha(iz)}{(iz)^\alpha}$ with $J_\alpha(z)$ the Bessel function of order $\alpha > -1$. Taking $\alpha = -1/2$ we get $\Lambda_{-1/2} = d/dx$ and $E_{-1/2}(z) = e^z$, hence, the Dunkl derivative and the Dunkl exponential are generalizations of the classic derivative and exponential, respectively. With this, in recent papers (see, for instance, [1] or [2]) there were generalized, in a Dunkl sense, Number Theory ideas such as Appell sequences, called Appell-Dunkl sequences. Our investigations leads to generalizations, also in a Dunkl sense, of Lerch zeta functions such as $\zeta(s,x) = \sum_{n=0}^\infty 1/(n+x)^s$ or $\zeta_E(s,x) = \sum_{n=0}^\infty (-1)^n/(n+x)^s$ and, in addition, we get properties that relates those functions with Bernoulli and Euler polynomials (in this case, they are called Bernoulli-Dunkl and Euler-Dunkl polynomials). One of the results we get for the "Dunkl zeta function", $\zeta_\alpha(s)$, is

$$\zeta_\alpha(1-s) = \Gamma(s)\cos\left(\frac{\pi s}{2}\right)\sum_{n=1}^\infty \frac{1}{s_n^s}, \qquad \mathrm{Re}(s) > 1$$

(where $s_n := s_{n,\alpha}$ are the positive zeros of the Bessel function $J_{\alpha+1}(x)$). Therefore, this equation provides a generalization of the reflection formula of the Riemann zeta function, where the function $\sum_{n=1}^\infty 1/s_n^s$ is playing a similar role as $\sum_{n=1}^\infty 1/n^s$.

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Universal quadratic forms and indecomposables in simplest cubic fields

Daniel Gil-Muñoz[1]

A quadratic form with coefficients in a totally real number field K is said to be universal if it represents all totally positive integers of K (that is, those that are positive after applying all embeddings of K). For instance, Lagrange's four square theorem assures that the rational quadratic form $x^2 + y^2 + z^2 + t^2$ is universal. An interesting question is how many unknowns are needed for a quadratic form with coefficients in K to be universal. From the work by Blomer and Kala [1, 2], we know that this question is closely linked with the study of additive indecomposables in the number ring $\mathcal{O}_K$ of K. In this talk we will review the work by Kala and Tinková [3] on the determination of indecomposables in Shanks' family of simplest cubic fields for the monogenic case and present a work in progress on some non-monogenic fields within this family.

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Parametrizations of isogeny-torsion graphs of elliptic curves over $\mathbb{Q}$

Enrique González Jiménez[1]

The isogeny graph of a $\mathbb{Q}$-isogeny class of elliptic curves defined over $\mathbb{Q}$ consists in a vertex for each elliptic curve in the isogeny class and an edge for each rational isogeny of prime degree between elliptic curves in the isogeny class, with the degree recorded as a label on the edge. The isogeny graphs of elliptic curves over $\mathbb{Q}$ first appeared in the so-called Antwerp tables [1]. Although the first proof (in press) seems to be due to Chiloyan and Lozano-Robledo [2, § 6]. In the first part of this talk we will show parametrizations of these isogeny graphs. Moreover, Chiloyan and Lozano-Robledo [2] define isogeny-torsion graph to be an isogeny graph where, in addition, each vertex is labeled with the abstract group structure of the torsion subgroup of the corresponding elliptic curve. They classify all the possible isogeny-torsion graphs that occur for $\mathbb{Q}$-isogeny classes of elliptic curves defined over $\mathbb{Q}$. In the last part of this talk we will show parametrizations of these isogeny-torsion graphs.

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Una de las últimas conjeturas de Javier Cilleruelo

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Dado un entero positivo n, cada divisor $d \mid n$ puede ser sumado con su correspondiente divisor complementario n/d, y haciendo esto con todos los divisores se obtiene el conjunto de números $S(n) := \{d + n/d : d \leq n\}$. Aunque este conjunto a primera vista puede parecer completamente aleatorio e irrelevante, observando los números que aparecen es posible apreciar que para algunos n ocurre un fenómeno aritmético curioso en sus respectivos $S(n)$: la presencia de números consecutivos. Aquí se intentará aclarar y formalizar el porqué de este fenómeno aritmético.

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A program to reconstructing points of superelliptic curves over a prime finite field in SageMath

Jaime Gutierrez[1]

We present a SageMath implementation and empirical results of the main algorithm provided in paper [1] for reconstructing points of superelliptic curves.

Here we consider the following computational problem: given the polynomial $Y^n + f(X) \in \mathbb{F}_p[X, Y]$ and approximations to $(v_0, v_1) \in \mathbb{F}_p^2$ where $v_1^n + f(v_0) \equiv 0 \bmod p$, reconstruct (v_0, v_1).

Its has applications to, and has been motivated by, the predictability problem for the linear congruential generator on elliptic and hyperelliptic curves.

This problem is a particular case of the problem of finding small solutions of multivariate polynomial congruences. All of them are based on the so called lattice reduction techniques.

References

[1] J. Gutierrez, Reconstructing points of superelliptic curves over a prime finite field, *Adv. Math. Commun.*, online first, 2022, doi: 10.3934/amc.2022022.

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Integrales zeta y modelos de Shalika para $\mathrm{GU}(2,2)$

Antonio Cauchi[1], **Armando Gutiérrez Terradillos**[2]

La relación entre periodos de formas automorfas y los polos de las funciones L de Langlands es uno de los temas más interesantes en la teoría de números. Dada una representación automorfa cuspidal π de un grupo G y una representación r del grupo de Langlands dual ${}^LG$, conjeturalmente se espera encontrar las siguientes relaciones:

1. La función $L(s, \pi, r)$ tiene un polo.
2. Cierto periodo asociado a la representación automorfa π es diferente de 0.
3. La representación automorfa π es un lift (débil) funtorial respecto a una aplicación inyectiva $H \hookrightarrow {}^LG$, siendo H el estabilizador de un punto genérico de la representación r.

En esta presentación explicaré la teoría básica de representaciones automorfas y enunciaré los resultados obtenidos en [1] para los grupos $\mathrm{GU}(2,2)$ y GSp_4.

Referencias

[1] A. Cauchi, A. Gutiérrez Terradillos, Zeta integrals and Shalika models for $\mathrm{GU}(2,2)$, *en progreso*, 2022.

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Computing the endomorphism ring of a supersingular elliptic curve

Jenny G. Fuselier[1], Annamaria Iezzi[2], Mark Kozek[3], Travis Morrison[4], Changningphaabi Namoijam[5]

In recent years, isogeny-based cryptosystems have captured the attention of the math/crypto community for their potential resistance to quantum attacks. In this context, the most promising protocols have as central objects supersingular elliptic curves defined over a finite field, and their security is therefore based on the mathematical problem of calculating an isogeny between two supersingular elliptic curves E and E'. It has been shown that this problem can be reduced to the calculation of the endomorphism rings of E and E'. In this talk, after reviewing the mathematical and cryptographic context, we will then present an improved algorithm for computing the endomorphism ring of a supersingular elliptic curve over a finite field.

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Plectic points and Hida-Rankin p-adic L-functions

Santiago Molina Blanco[1]

Plectic points were introduced by Fornea and Gehrmann as certain tensor products of local points on elliptic curves over arbitrary number fields F. In rank $r < [F : \mathbb{Q}]$-situations, they conjecturally come from p-adic regulators of basis of the Mordell-Weil group defined over dihedral extensions of F.

In the project I want to discuss we define two variable anticyclotomic p-adic L-functions attached to a family of overconvergent modular symbols defined over F and a quadratic extension K/F. Their restriction to the weight space provide Hida-Rankin p-adic L-functions.

If such a family passes through an overconvergent modular symbol attached to a modular elliptic curve E, we obtain a p-adic Gross-Zagier formula that computes higher derivatives of such Hida-Rankin p-adic L-functions in terms of plectic points. This result generalizes that of Bertolini and Darmon, which has been key to demonstrating the rationality of Darmon points.

This is a joint work with Víctor Hernández Barrios.

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$-\log 2$ is a key number for the Dirichlet series

Gaspar Mora[1], **Edgar R. Benítez**[2]

In this communication it is shown that $-\log 2$ is a number that plays a crucial role in the Dirichlet series theory. Indeed, it will be demonstrated that given any Dirichlet series

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^s}, \quad s = \sigma + it \in \mathbb{C},$$

with $f(n)$ an arbitrary arithmetical function of modulus 1 and $f(1) = 1$, one has

$$-\log 2 \leq \frac{a_n}{n} \text{ for all } n > 1,$$

where

$$a_n := \inf \left\{ \sigma : \sum_{k=1}^{n} \frac{f(k)}{k^s} = 0 \right\},$$

i.e. a_n, $n > 1$, is the lower bound of the real part of the zeros of the nth partial sum of the Dirichlet series given.

Furthermore if f is completely multiplicative, then there exists $\lim_n a_n/n$ and

$$\lim_n \frac{a_n}{n} = -\log 2$$

holds. Consequently this relation is true for the Riemann zeta series.

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Overconvergent F-isocrystals as de Rham–Witt connections

Rubén Muñoz-Bertrand[1]

Rigid cohomology can be used to evaluate the Weil zeta function of an algebraic variety of positive characteristic. To get more general L-functions, one can use a category of coefficients called overconvergent F-isocrystals. In this communication, we will explain how overconvergent F-isocrystals can be described as connections on a module over a Zariski sheaf when the variety is smooth over a perfect field, as envisioned by Ertl [2]. To do this, we shall employ the overconvergent de Rham–Witt complex, which has recently been introduced as a mean to compute rigid cohomology [1].

We will recall the basic notions used in Monsky–Washnitzer cohomology [3] to the audience, and explain how the concept of weak completion can be translated in the context of the de Rham–Witt complex. We will then evoke how this enables one to do p-adic analysis with these tools.

References

[1] C. Davis, A. Langer, T. Zink, Overconvergent de Rham–Witt cohomology, *Ann. Sci. Éc. Norm. Supér.* **44** (2011), 197–262.

[2] V. Ertl, Full faithfulness for overconvergent F-de Rham–Witt connections, *C. R. Math. Acad. Sci. Paris* **354** (2016), 653–658.

[3] P. Monsky, G. Washnitzer, Formal cohomology. I, *Ann. of Math. (2)* **88** (1968), 181–217.

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Leyes de reciprocidad intrínsecas

(Al Dr. José María Muñoz Porras, In Memoriam)

Fernando Pablos Romo[1]

La charla está destinada a analizar leyes de reciprocidad que pueden probarse de forma intrínseca a partir de ideas del Prof. Dr. José María Muñoz Porras. Así, se ofrecerá la formulación abstracta de leyes clásicas de reciprocidad (entre ellas la ley de reciprocidad de Weil) que fue demostrada en [1]. Y también se presentarán los resultados fundamentales de [2], donde se ofreció una ley de reciprocidad asociada con revestimientos finitos de curvas algebraicas, que permite dar ejemplos concretos de leyes de reciprocidad en curvas algebraicas que no pueden deducirse de la ley de reciprocidad de Weil.

Referencias

[1] J. M. Muñoz Porras, F. Pablos Romo, Generalized reciprocity laws, *Trans. Amer. Math. Soc.* **360** (2008), 3473–3492

[2] J. M. Muñoz Porras, F. Pablos Romo, F. J. Plaza Martín, An explicit reciprocity law associated to some finite coverings of algebraic curves, *Mediterr. J. Math.* **15** (2018), Paper No. 82, 18 pp.

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Sobre el grupo de 2-Selmer de curvas elípticas

Daniel Barrera Salazar[1], **Ariel Pacetti**[2], **Gonzalo Tornaría**[3]

Sea E una curva elíptica definida sobre un cuerpo de números K dada por una ecuación de la forma

$$E : y^2 = f(x),$$

donde $f(x)$ es un polinomio irreducible con coeficientes en el anillo de enteros de K. En esta charla, fruto de [1], mostraremos cómo acotar el grupo de 2-Selmer de la curva elíptica en términos de la 2-torsión del grupo de clases de $K(E[2])$. A la vez, mostraremos algunas aplicaciones al estudio de variación del rango del grupo de 2-Selmer en familias de twists cuadráticos.

Referencias

[1] D. Barrera Salazar, A. Pacetti, G. Tornaría, On 2-Selmer groups and quadratic twists of elliptic curves, *Math. Res. Let.*, por aparecer.

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Some inverse problems on sumsets

Ram Krishna Pandey[1]

Let $h \geq 2$ be an integer and A be a non-empty finite set of integers. The *h-fold sumset* of A, denoted by hA, is the set of all integers that are representable as a sum of h elements of A and the *restricted h-fold sumset* of A, denoted by $h\hat{}A$, is the set of all integers that are representable as a sum of h distinct elements of A. One of the fundamental problems in additive number theory is to estimate the size of the sumsets hA and $h\hat{}A$ and conversely, characterize the sets A for which the sizes of these sumsets is minimum. Various types of sumsets are known now (which are the generalizations of these two fundamental sumsets) such as the generalized h-fold sumset [3], signed sumsets [2], and sumsets of dilated sets [1]. We study inverse problems for some of these sumsets in this talk with some early and recent generalizations.

References

[1] J. Bhanja, S. Chaudhary, R. K. Pandey, On some direct and inverse results concerning sums of dilates, *Acta Arithmetica* **188** (2019), 101–109.

[2] J. Bhanja, R. K. Pandey, Direct and inverse theorems on signed sumsets of integers, *J. Number Theory* **196** (2019), 340–352.

[3] R. K. Mistri, R. K. Pandey, A generalization of sumsets of set of integers, *J. Number Theory* **143** (2014), 334–356.

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Kummer theory for number fields

Antonella Perucca[1], Pietro Sgobba[1], Sebastiano Tronto[1]

We give an overview of the results on Kummer theory for number fields by the authors joint with Christophe Debry (KU Leuven) and Fritz Hörmann (Freiburg). The focus is computing the degrees for families of cyclotomic-Kummer extensions. If K is a number field and G is a finitely generated subgroup of $K^\times$, then we investigate the cyclotomic-Kummer extension $K(\zeta_{nm}, \sqrt[n]{G})/K$ for all positive integers n and m.

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Left braces of size $8p$

Teresa Crespo[1], Daniel Gil-Muñoz[2], Anna Rio[3], Montserrat Vela[3]

We describe all left braces of size $8p$ for an odd prime $p \neq 3, 7$ and validate the number given by Bardakov, Neschadim and Yadav in [1]. We give a characterization for isomorphism classes of a semidirect product of left braces and then the description is done by first describing left braces of size 8, as conjugacy classes of regular subgroups of the corresponding holomorph, and then checking how many non isomorphic left braces of size $8p$ are obtained from each one of them.

References

[1] V. G. Bardakov, M. V. Neshchadim, M. K. Yadav, Computing skew left braces of small orders, *Internat. J. Algebra Comput.* **30** (2020), 839–851.

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Sistemas de Euler anticiclotómicos y ciclos diagonales I

Raúl Alonso[1], **Francesc Castellà**[2], **Óscar Rivero**[3]

El estudio de los sistemas de Euler ha permitido obtener en los últimos años resultados en torno a las conjeturas de Birch y Swinnerton-Dyer, Bloch–Kato o la conjetura principal de Iwasawa. En esta teoría, los denominados sistemas anticiclotómicos desempeñan un papel preponderante, dado que se obtienen de forma natural en el estudio de las variedades algebraicas sobre cuerpos cuadráticos imaginarios.

En esta presentación repasaré algunos de los últimos avances en la teoría de los sistemas de Euler y discutiré la motivación detrás de dos trabajos conjuntos con Raúl Alonso y Francesc Castellà, en los que construimos sistemas de Euler para la convolución de dos formas modulares y para el cuadrado simétrico de una forma modular, usando para ello familias p-ádicas de ciclos diagonales. Esto nos permite obtener aplicaciones en el estudio de la conjetura de Bloch–Kato cuando el rango analítico es cero o uno, y también deducir una divisibilidad en la conjetura principal de Iwasawa.

Esta presentación se basa en los trabajos [1] y [2].

Referencias

[1] R. Alonso, F. Castellà, Ó. Rivero, The diagonal cycle Euler system for $\mathrm{GL}_2 \times \mathrm{GL}_2$, *preprint*, 2021.

[2] R. Alonso, F. Castellà, Ó. Rivero, An anticyclotomic Euler system for the symmetric square of a modular form, *preprint*, 2022.

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Motivos hipergeométricos

David P. Roberts[1]

El artículo [1] describe una construcción general que comienza en un vector de números enteros, pasa por una familia de variedades algebraicas y termina en una colección de funciones L. El interés en esta construcción es que las funciones L obtenidas son una muestra representativa de todas las funciones L motívicas y, al mismo tiempo, permiten cálculos numéricos de alta precisión. La charla dará una introducción a esta construcción mediante un ejemplo relativamente simple. Este ejemplo comenzará desde $[-5, -2, 3, 4]$, recorrerá una familia de curvas de género dos y terminará en cálculos de los ceros de algunas funciones L.

Referencias

[1] D. P. Roberts, F. Rodríguez Villegas, Hypergeometric motives, *Notices Amer. Math. Soc.*, por aparecer, June/July 2022.

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Fundamental domains for the Bruhat-Tits tree for $\mathrm{GL}_2(F_{\mathfrak{p}})$

Eloi Torrents Juste[1]

The computation of fundamental domains of the Bruhat-Tits tree by the action of quaternionic groups allows the computation of harmonic cocycles on it. These are related to automorphic forms and from this fact are derived several applications, as for example the computation of points on Shimura curves and Heegner points on elliptic curves as done by M. Greenberg and later generalized by C. Franc and M. Masdeu.

In this talk we will review these concepts, and we will explain how to apply them in the computation of Heegner points on elliptic curves in cases where the Heegner hypothesis is not satisfied, and therefore the classical arquimedian construction of these points is difficult to compute.

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Sign changes of the Liouville function on quadratics

Anitha Srinivasan[1]

Are there infinitely many primes of the kind $n^2 + 1$? Or, more generally, is $n^2 + d$ prime for infinitely many n? We all believe this is true but a proof still eludes us. Even the simpler question that asks whether $n^2 + d$ has an odd number of primes infinitely often, is a challenging one. Existing results answer this question only conditionally. In this talk we present our result that settles this case: we show that there are infinitely many n such that $n^2 + d$ has an odd number of prime divisors (counting multiplicity). The analogous result for an even number of primes also holds. The proof is based on the theory of genera of binary quadratic forms.

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Horario / *Schedule*

Dos charlas, marcadas con (C), fueron canceladas por culpa del covid. / *Two talks, marked with (C), were cancelled due to the covid.*

Martes 28 de junio / *June 28th, Tuesday*

- 9:15–19:30 Entrega de documentación / *Delivery of documentation*
- 9:30–10:00 Inauguración / *Opening*
- 10:00–11:00 **P. Bayer**: On Shimura curves of PEL-type
- 11:00–11:30 Café / *Coffee*
- 11:30–12:00 **A. Pacetti**: Sobre el grupo de 2-Selmer de curvas elípticas
- 12:00–12:30 **A. Perucca**: Kummer theory for number fields
- 12:30–13:00 **J. Granados**: Una de las últimas conjeturas de Javier Cilleruelo

- 13:00–13:30 **A. Rio**: Left braces of size $8p$
- 13:30 Comida / *Lunch*
- 15:30–16:30 **S. Anni**: Counting modular forms with a Galois representation mod p and the Atkin-Lehner eigenvalue at p fixed simultaneously
- 16:30–17:00 **R. K. Pandey**: Some inverse problems on sumsets
- 17:00–17:30 Café / *Coffee*
- 17:30–18:00 **D. Gil-Muñoz**: Universal quadratic forms and indecomposables in simplest cubic fields
- 18:00–18:30 **Ó. Rivero**: Sistemas de Euler anticiclotómicos y ciclos diagonales I
- 18:30–19:00 **R. Alonso**: Sistemas de Euler anticiclotómicos y ciclos diagonales II

Miércoles 29 de junio / *June 29th, Wednesday*

- 9:30–10:30 **M. Masdeu**: Computing plectic Stark-Heegner invariants
- 10:30–11:00 **S. Molina**: Plectic points and Hida-Rankin p-adic L-functions
- 11:00–11:30 Café / *Coffee*
- 11:30–12:00 **M. I. de Frutos**: Formalizing the ring of adèles and some applications in Lean
- 12:00–12:30 **D. P. Roberts**: Motivos hipergeométricos
- 12:30–13:00 **F. Bars**: On infinity of cubic points over the rationals for quotient modular curves
- 13:00–13:30 **T. Dalal**: On Drinfeld oldforms and newforms
- 13:30 Comida / *Lunch*
- 15:30–16:30 **L. Thompson**: Sumar $\mu(n)$: un algoritmo elemental más rápido
- 16:30–17:00 **A. Gutiérrez Terradillos**: Integrales zeta y modelos de Shalika para $\mathrm{GU}(2,2)$
- 18:30 Excursión / *Excursion* (Bodegas Franco-Españolas)

Jueves 30 de junio / *June 30th, Thursday*

- 9:30–10:30 (C) **V. Cantoral-Farfán**: Sobre el grupo de componentes del grupo de Sato-Tate
- 10:30–11:00 **P. J. Cazorla**: The modular method and its limitations
- 11:00–11:30 Café y foto de grupo / *Coffee and group photo*
- 11:30–12:00 **E. Torrents**: Fundamental domains for the Bruhat-Tits tree for $\mathrm{GL}_2(F_\mathfrak{p})$
- 12:00–12:30 **A. Srinivasan**: Sign changes of the Liouville function on quadratics
- 12:30–13:00 **R. Muñoz-Bertrand**: Overconvergent F-isocrystals as de Rham–Witt connections
- 13:00–13:30 **F. Pablos**: Leyes de reciprocidad intrínsecas
- 13:30 Comida / *Lunch*

- 15:30–16:30 **H. Darmon**: Rigid meromorphic cocycles
- 16:30–17:00 **A. Gil Asensi**: On Dunkl zeta functions and their properties
- 17:00–17:30 Café / *Coffee*
- 17:30–18:00 **J. Gutierrez**: A program to reconstructing points of superelliptic curves over a prime finite field in SageMath
- 18:00–18:30 **A. Iezzi**: Computing the endomorphism ring of a supersingular elliptic curve
- 18:30–19:00 Reunión 10JTN / *10JTN meeting*
- 21:00 Cena / *Dinner*

Viernes 1 de julio / *July 1st, Friday*

- 9:30–10:00 **G. Mora**: $-\log 2$ is a key number for the Dirichlet series
- 10:00–10:30 **E. Florit**: Abelian varieties of GL_4 type
- 10:30–11:00 (C) **D. Martínez (MIdF)**: Towards a refined class number formula for Drinfeld modules
- 11:00–11:30 Café / *Coffee*
- 11:30–12:00 **M. Bello-Hernández**: La conjetura de Erdős-Straus
- 12:00–12:30 **E. González**: Parametrizations of isogeny-torsion graphs of elliptic curves over $\mathbb{Q}$
- 12:30–13:30 **P. Candela**: On a generalization of Fourier analysis arising from number theory
- 13:30–13:45 Clausura / *Closing*
- 13:45 Comida / *Lunch*

Participantes / *Participants*

- Raúl Alonso Rodríguez (Princeton University)
- Samuele Anni (Aix-Marseille Université)
- Alberto Arenas (Universidad de La Rioja)
- Francesc Bars (Universitat Autònoma de Barcelona)
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- María Chara (CONICET, Argentina)
- Capi Corrales Rodrigáñez (Universidad Complutense de Madrid)
- Tarun Dalal (Indian Institute of Technology Hyderabad)

Foto de grupo / *Photo of the assistants*

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