On algebraic properties of the human ABO-blood group inheritance pattern

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Joint work with J. M. Casas, M. Ladra, B. A. Omirov

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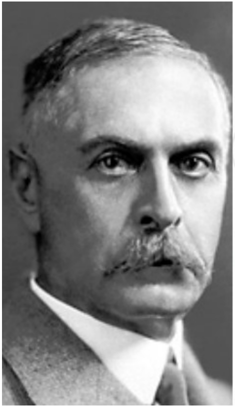
Unión Europea
Fondo Europeo de Desarrollo Regional

Karl Landsteiner 148th birthday on 14th of June



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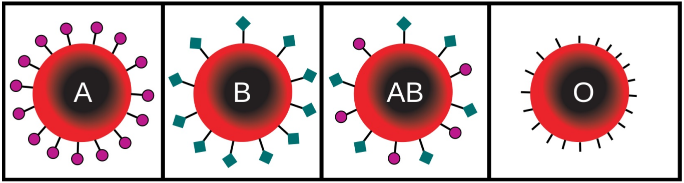
ABO-blood group system

Presence and absence of two different types of agglutinogens, type "A" and type "B" determines four major ABO-blood groups.



Short definitions of some terms of genetics

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Example

The gene which determines blood group in humans has three different alleles :

A, B, and O.

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- There are four blood groups (phenotypes): O, A, B and AB.

Establishing the genetics of ABO-blood groups

From parents to child...

- When diploid organisms reproduce, a process called meiosis produces gametes (sex cells) which carry a single set of chromosomes.

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- If parents are of blood group O and AB possibilities for zygotes are A or B only.

Probabilities of zygotes' blood groups

Parents	A	B	O	AB
A	A (15/16) O (1/16)	A (3/16) B (3/16) O (1/16) AB (9/16)	A (3/4) O (1/4)	A (1/2) B (1/8) AB (3/8)
B	A B O AB	B (15/16) O (1/16)	B (3/4) O (1/4)	A (1/8) B (1/2) AB (3/8)
O	A O	B O	O (1)	A (1/2) B (1/2)
AB	A B AB	A B AB	A B	A (1/4) B (1/4) AB (1/2)

Here it is assumed that parents' gametes are chosen randomly and independently during meiosis.

When gametes fuse to form zygotes a natural multiplication happens.

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The systematic study of algebras occurring in genetics was due to **I. M. H. Etherington**. Presented a precise mathematical formulation of Mendel's laws in terms of *non-associative algebras*.

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Consider the blood groups O, A, B and AB as basis elements of a four-dimensional vector space over $\mathbb{R}$ and a bilinear operation $\circ$ as the result of meiosis.

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- A and B have equal probabilities to contribute with the allele O to a child's genotype $p_{O|A} = p_{O|B} = \alpha$.
- All parents with group AB contribute the allele A during meiosis with probability $p_{A|AB} = \beta$.

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Definition

A commutative four-dimensional $\mathbb{R}$-algebra with basis $\{O, A, B, AB\}$ and with multiplication $\circ$ satisfying equalities (i)–(x) is called a generalized ABO-blood group algebra (GBGA) and is denoted by $\mathcal{B}(\alpha, \beta)$.

Generalized ABO-blood group algebra $\mathcal{B}(\alpha, \beta)$

(i) $O \circ O = O$;
(ii) $O \circ A = \alpha O + (1-\alpha)A$;
(iii) $O \circ B = \alpha O + (1-\alpha)B$;
(iv) $O \circ AB = \beta A + (1-\beta)B$;
(v) $A \circ A = \alpha^2 O + (1-\alpha^2)A$;
(vi) $A \circ B = \alpha^2 O + \alpha(1-\alpha)A + \alpha(1-\alpha)B + (1-\alpha)^2 AB$;
(vii) $A \circ AB = \beta A + \alpha(1-\beta)B + (1-\alpha)(1-\beta)AB$;
(viii) $B \circ B = \alpha^2 O + (1-\alpha^2)B$;
(ix) $B \circ AB = \alpha\beta A + (1-\beta)B + (1-\alpha)\beta AB$;
(x) $AB \circ AB = \beta^2 A + (1-\beta)^2 B + 2\beta(1-\beta)AB$.

Here $\alpha = p_{O|A} = p_{O|B}$ and $\beta = p_{A|AB}$.

More assumptions

We assume that $0 < \alpha, \beta < 1$.

Remark

Note that if we interchange A and B and β to $1 - \beta = 1 - p_{A|AB} = p_{B|AB}$, we obtain the same products as above. That is $\mathcal{B}(\alpha, \beta) \cong \mathcal{B}(\alpha, 1 - \beta)$.

Algebraic relations of GBGA from a different perspective

Let x_1, x_2, x_3, x_4 be corresponding proportions of O, A, B, AB blood groups in one population. Then the underlying allele frequencies are equal to

$$\begin{aligned} p_O &= x_1 + \alpha x_2 + \alpha x_3, \\ p_A &= (1-\alpha)x_2 + \beta x_4, \\ p_B &= (1-\alpha)x_3 + (1-\beta)x_4. \end{aligned}$$

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The frequencies of O, A, B and AB phenotypes in zygotes of the next generation (state) verifies classical Hardy-Weinberg Law:

$$\begin{cases} x_1' = p_O^2 \\ x_2' = p_A^2 + 2p_A p_O \\ x_3' = p_B^2 + 2p_B p_O \\ x_4' = 2p_A p_B. \end{cases}$$

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Evolutionary map: $\mathbf{x} = (x_1, x_2, x_3, x_4) \mapsto (x_1', x_2', x_3', x_4') = V(\mathbf{x})$

Evolutionary operator and multiplication ∘

Evolutionary operator and multiplication $\circ$

The relation that establishes a connection between the evolutionary operator V and the multiplication $\circ$ of a GBGA is

$$\mathbf{x} \circ \mathbf{x} = V(\mathbf{x})$$

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and consequently

$$\mathbf{x} \circ \mathbf{y} = \frac{1}{4}(V(\mathbf{x}+\mathbf{y}) - V(\mathbf{x}-\mathbf{y})).$$

Consider the dynamics of $\mathbf{x}, V(\mathbf{x}), V^2(\mathbf{x}), \dots.$ for a given population $\mathbf{x}$.

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Population (or equivalently, an element in our algebra) $\mathbf{x}$ is called idempotent if $\mathbf{x} = V(\mathbf{x})$.

In order to simplify our investigation of the structure of a GBGA we make the following linear basis transformation

$$\begin{cases} o = O \\ a = \dfrac{1}{(1-\alpha)^2}(O - A) \\ b = \dfrac{1}{(1-\alpha)^2}(O - B) \\ ab = \dfrac{1}{(1-\alpha)^3}(\alpha O - \beta A - (1-\beta)B + (1-\alpha)AB) \end{cases}$$

and obtain a simpler table of multiplication of a GBGA.

After basis transformation...

Algebra $\mathcal{B}(\alpha, \beta)$ in the basis $\{o, a, b, ab\}$ admits the following non-zero table of multiplication

$$\mathcal{B}'(\lambda, \beta) : \begin{cases} o \circ o = o \\ o \circ a = a \circ o = \lambda a \\ o \circ b = b \circ o = \lambda b \\ a \circ a = a \\ b \circ b = b \\ a \circ b = b \circ a = \dfrac{\lambda - \beta}{\lambda} \cdot a + \dfrac{\lambda - (1 - \beta)}{\lambda} \cdot b + ab, \end{cases}$$

where $\lambda = 1 - \alpha$ and omitted products are assumed to be zero.

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Proposition

The only absolute nilpotent element of $\mathcal{B}'(\lambda, \beta)$ up to scalar multiple is ab.

Set of dominating subpopulations (lattice of ideals)

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$\mathcal{B}'(\lambda, \lambda), \lambda \neq \frac{1}{2}:$

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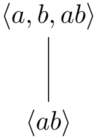

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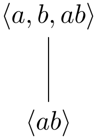
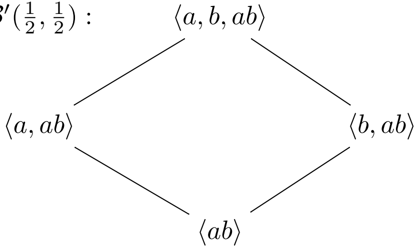


$\mathcal{B}'(\frac{1}{2}, \frac{1}{2}) :$



Idempotents elements

Denote by $P = \left\{(\lambda, \beta) \mid 0 < \lambda \leq \frac{1}{3},\ \beta = \frac{1}{2}\left(1 \pm \sqrt{(1-\lambda)(1-3\lambda)}\right)\right\}$.

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- $\mathcal{I} = \{o,\ a,\ b,\ o + (1-2\lambda)a,\ o + (1-2\lambda)b\}$ *if* $(\lambda, \beta) \in P \cup \{(2\beta, \beta) | \beta \neq \frac{1}{4}\} \cup \{(2-2\beta, \beta) | \beta \neq \frac{3}{4}\}$;

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- $\mathcal{I}=\{o,\ a,\ b,\ o+(1-2\lambda)a,\ o+(1-2\lambda)b\}$ *if* $(\lambda,\beta)\in P\cup\{(2\beta,\beta)|\beta\neq\frac{1}{4}\}\cup\{(2-2\beta,\beta)|\beta\neq\frac{3}{4}\}$;
- $\mathcal{I}=\{o,\ a,\ b,\ o+(1-2\lambda)a,\ o+(1-2\lambda)b,\ j_0,\ j_1\}$, *otherwise,*

where $j_\xi=\xi o+\rho_\xi(2\beta-\lambda)a+\rho_\xi(2-2\beta-\lambda)b+2\rho_\xi^2(2\beta-\lambda)(2-2\beta-\lambda)ab$ *and*
$\rho_\xi=\dfrac{\lambda(1-2\xi\lambda)}{-3\lambda^2+4\beta^2+4\lambda-4\beta}$ *for* $\xi=0,1$.

Solvable elements

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Moreover, solvable elements of degree n are

$$-2^{n-4}\left(\frac{\lambda+\beta-1}{\lambda}\right)^{n-4} ta + tb + sab, \quad \textit{where} \ \ t, s \in \mathbb{R},\ t \neq 0.$$

Inheritance pattern for different values of parameters

Theorem

Two distinct ABO-blood group algebras $\mathcal{B}'(\lambda, \beta)$ and $\mathcal{B}'(\lambda', \beta')$ are isomorphic if and only if $\lambda' = \lambda$ and $\beta' = 1 - \beta$.

Results are presented in

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Thank you!