Developments in RUBI: RUle-Based Integration

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Overview

- The Rubi project
- From linear search to tree search.
- Rubi reaches Maple
- A quick tour of the web site.

The Project

- RUBI has 2 aims: to promote rule-based mathematical software; to integrate functions.
- The second aim, integrating functions, helps the first by providing 'proof of concept'.
- The immediate (short-term) aim of RUBI is to create a public-domain database of rules for the evaluation of indefinite integrals, also called anti-derivatives or primitives.
- In addition to the integration rules, an important component is a test suite containing integration problems with solutions.
- The current version of RUBI is freely available from `www.apmaths.uwo.ca/~arich`
- The originator and driving force of the project is Albert Rich.

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The main author

Albert wrote muLisp, muMath and Derive before starting work on Rubi.

But MAPLE and MATHEMATICA can already integrate

- Yes they can, but RUBI does it better.
- In the context of integration, Rubi aims to get OPTIMAL primitives, or anti-derivatives, not just any primitive.
- It does this efficiently and effectively using a rule-based system, also called rewrite system.

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Some statistics

- The test suite contains over $55,000$ items. Each item has been checked for correctness, generality and optimality. It is becoming an item of independent interest.
- The database of evaluation rules contains over 6000 rules.

What is optimal?

There are several measures of optimal.

Expression size: Maple demonstration 1: optimal

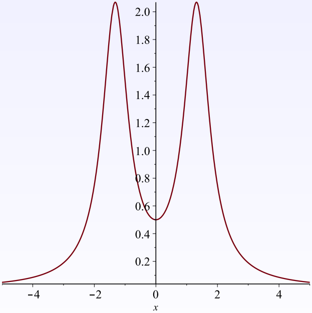
$$\int \frac{x^{10}}{(1+x)^{12}}\,\mathrm{d}x = \text{Take your pen and paper ...}$$

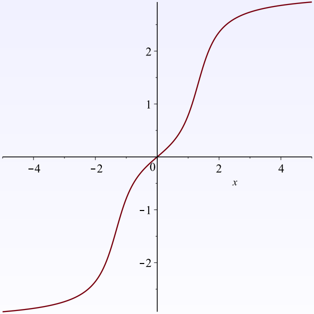
Another aspect of Optimal

Consider the function

$$f = \frac{x^2 + 2}{x^4 - 3x^2 + 4}\ .$$

Its graph and integral are

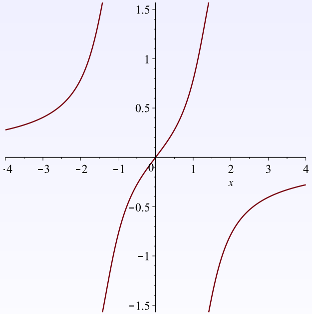




But many books say ...

$$\int \frac{(x^2+2)\,\mathrm{d}x}{x^4-3x^2+4} = \operatorname{arctan} \frac{x}{2-x^2}\ ,$$

which has the graph



The continuous primitive

An expression which is continuous for all x is

$$\int \frac{(x^2+2)\,\mathrm{d}x}{x^4-3x^2+4} = \arctan(2x+\sqrt{7}) + \arctan(2x-\sqrt{7})\ .$$

This form is obtained by Rubi and Maple, but the discontinuous form is preferred by Mathematica.

Some years ago, the late Manuel Bronstein said he could not put continuous forms in Axiom because customers would not understand.

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The Customer is Always Right (an aside)

Commercial computer algebra systems have to balance the desires of customers and mathematical correctness.
For example, system developers want

$$\int \frac{dx}{x} = \ln x \ ,$$

while many customers (school teachers) demand

$$\int \frac{dx}{x} = \ln |x| \ .$$

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Another criterion: Æsthetics

Consider

$$\int \frac{16}{16 - x^4}\,\mathrm{d}x = \frac{1}{2}\ln(2+x) - \frac{1}{2}\ln(x-2) + \arctan(x/2)$$
$$= \arctan(x/2) + \mathrm{arctanh}(x/2)$$

Why so many rules?

- Special cases.
- Efficiencies
- Algebraic simplifications

Implementation

- Initially developed and implemented in MATHEMATICA.
- MATHEMATICA has the strongest pattern matching functions.
- The program consisted of stepping through the list of rules, and for each rule
  1. Call the pattern matcher to decide whether this rule fits the integrand.
  2. Check the values of the parameters against the applicability conditions.
  3. Apply the rule if successful and recursively call the program.
  4. If not successful, step to next rule.

Weaknesses

- The search is linear.
- Because each rule is independent, it is difficult to keep track of cases during development. Have all cases been covered? Is there overlap?
- Only Mathematica has a strong pattern matcher. Implementing in Maple (for example) relying on `patmatch`, `type` is frustrating.

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New implementation

- Convert to binary tree search.
    1. More efficient (as every computer scientist knows).
    2. Also imposes order on the development. Missing cases are easier to identify.
- Write separate utility functions for MAPLE and other systems, to remove reliance on system pattern matching.