Moving frames and Noether's Theorem: Smooth and discrete case.

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POSTER ABSTRACT

One of the most famous problems studied by Euler is to find the curves $(x, y(x))$ that minimise the integral

$$\int \kappa^2 ds, \tag{1}$$

where $\kappa = \frac{y_{xx}^2}{(1+{y_x}^2)^{\frac{3}{2}}}$ is the curvature and $ds = \sqrt{1+{y_x}^2}\,dx$ is the derivative of the arc length.

Euler showed in [1] that these minimising curves satisfy the differential equation

$$\kappa_{ss} + \frac{1}{2}\kappa^3 = 0.$$

This equation is the Euler-Lagrange equation for (1) and the solutions of this equation are commonly known as Euler's elastica.

In this poster we show how the theory of moving frames can be used in order to solve the approximate variational problem of minimising the discrete analogous of (1) using Lie symmetries and Noether's Theorem. We present the relevant discrete Noether's theorem and we compare both smooth and discrete cases.

Applications: Animation, computer graphics and impainting problems.

[1] Euler, L., Methodus Inveniendi Lineas Curvus, Lausanne, 1744, Opera Omnia; ser. 1, vol. 24, Füssli, Zurich, 1960.