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Attitude stabilization of electrodynamic tethers in elliptic orbits by time-delay feedback control

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ABSTRACT

It is well known that libration motion of electrodynamic tethers operating in inclined orbits is affected by dynamic instability due to the electromagnetic interaction between the tether and the geomagnetic field. We study the application of two feedback control methods in order to stabilize the periodic attitude motions of electrodynamic tethers in elliptic inclined orbits. Both control schemes are based on the time-delayed autosynchronization of the system. Numerical simulations of the controlled libration motion show that both control techniques are able to transform the uncontrolled unstable periodic motions into asymptotically stable ones. Such stabilized periodic attitude motions can be taken as starting points for the operation of the tether. The control domains of both methods have been computed for different values of the system parameters, as functions of the two control parameters shared by both control schemes. The relative effectiveness of the two techniques in the stabilization of the periodic attitude motion has also been studied.

1. Introduction

Electrodynamic tether (EDT) satellite systems are a space technology that has been the object of wide research and development in the last decades [1,2]. From a theoretical point of view, these systems offer quite interesting models and problems in the field of non-linear dynamics. EDT satellites consist basically of two masses separated by a long conductive wire. When the tether is in orbit around the Earth, moving through the ionosphere, the tether can exchange free electrons with the ionospheric plasma. The electrons are collected at one end of the tether, and are then ejected from the other end. The conductive plasma closes the circuit, so that a electric current flows through the wire. The interaction of the current flowing through the tether with the geomagnetic field provides an electrodynamic force acting on the tether. This electromagnetic force may affect both the attitude and orbital dynamics of the EDT satellite.

The dynamics of EDT systems is a very interesting topic in space engineering and astrodynamics because it offers a variety of practical applications in spaceflight without the use of chemical or nuclear power sources. In this way, EDT spacecraft may provide means of satellite pointing, generation of electrical power, orbital debris mitigation and removal, as well as thrust or drag to perform orbital maneuvers. All of these EDT abilities are directly related to the orientation and attitude motion of the tether. Hence, during the last few years, many research efforts have been performed in order to understand the attitude dynamics, and control the libration motions of the EDT satellites [2].

It is well known that when an inert EDT, that is with zero current, describes a circular orbit, the system has stable equilibrium positions relative to the orbital reference frame. In these stable equilibrium positions, the tether is aligned with the local vertical due to the gravity gradient.

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Nomenclature

$\boldsymbol{A}(t)$	generic vectorial function
$\boldsymbol{B}$	(B_x, B_y, B_z), Earth's magnetic field in the orbital frame, T
e	eccentricity of the orbit
$EXYZ$	inertial geocentric frame
F, F_θ, F_φ	feedback control signals
$\boldsymbol{F}_{Lor}$	Lorentz force acting on the tether, N
G	system center of mass
$Gxyz$	orbital frame
h	orbital angular momentum, kg m^2/s
I	tether electric current, A
$\mathbb{I}$	diag$(0, I_y, I_z)$, inertia tensor of the tether in the body frame, kg m^2
i	orbital inclination, rad
k, k_θ, k_φ	feedback control gains
L	tether length, m
m	$m_1+m_2+m_t$, total mass of the system, kg
m_t	tether mass, kg
m_1	lower-end mass, kg
m_2	upper-end mass, kg
m^*	reduced mass of the system, kg
$Q_{c\,\theta}, Q_{c\,\varphi}$	control generalized forces, N
$Q_{ed\,\theta}, Q_{ed\,\varphi}$	electrodynamic generalized forces, N
R, R_θ, R_φ	parameters of the extended control method
$\boldsymbol{R}_{te}$	position vector of a tether element with respect to E, m
r	orbital radius of the tether, m
s	position coordinate along the tether from G, m
t	time, s
$\hat{\boldsymbol{u}}$	unit vector along the tether
ε	electrodynamic parameter
θ	tether in-plane libration angle, rad
μ_g	Earth's gravitational constant, m^3/s^2
μ_m	Earth's magnetic dipole moment strength, T m^3
ν	true anomaly, rad
τ	feedback control delay
φ	tether out-of-plane libration angle, rad
Ω	right ascension of the ascending node of the orbit, rad
$\boldsymbol{\omega}$	tether angular velocity, rad/s
$\tilde{\omega}$	argument of perigee of the orbit, rad

Superscripts

$(\dot{\star})$	$d(\star)/dt$
$(\star')$	$d(\star)/d\nu$

These stable equilibrium positions disappear when the tether describes an elliptic orbit. In this case, the satellite attitude dynamics shows libration periodic motions instead of those stable equilibrium positions [3–5].

When current is flowing through a tether that follows an inclined circular or elliptic orbit, the stable equilibrium positions along the local vertical also disappear and the tether attitude dynamics becomes unstable. The source of the instability is the electromagnetic interaction with the magnetic field of the Earth. This interaction provides a dynamical mechanism of energy interchange in the libration dynamics of the tether [6]. Under the assumption of a constant uniform current through the tether, in the case of inclined orbits, the attitude dynamics of the system exhibits unstable periodic libration motions instead of stable equilibrium positions aligned with the local vertical. Some of these periodic attitude motions as well as their stability properties have already been studied in the cases of circular and elliptic orbits [8,7]. Unfortunately, in the absence of damping or control, all these periodic libration motions are unstable, so that even small initial oscillation attitude motions become rotations in the long-term operation of the tether.

In recent decades, a great variety of new control methods has been developed to be applied to dynamical systems in order to transform unstable or chaotic motions into regular or periodic ones [9]. Delayed feedback control techniques constitute an interesting group among the current available control methods. One advantage these control schemes offer is that, in general, they need smaller control forces than non-feedback techniques in order to achieve the control of the system dynamics. In this sense, Pyragas [10] proposed a feedback control scheme designed to synchronize the current state of a system and a time delayed version of itself. Taking this delayed time as the period of an unstable periodic orbit, such a control scheme can be used to stabilize the orbit. This method of control is usually named time-delayed autosynchronization (TDAS). Two important advantages of this method are related to the feedback used: it does not require rapid switching or sampling, nor does it require a reference signal corresponding to the desired orbit. The TDAS technique has been improved by Socolar et al. [11], Pyragas [12], and Bleich and Socolar [13] using a more elaborated feedback, the so-called extended time-delayed autosynchronization (ETDAS), where TDAS appears as a limiting case. Frequently, dynamical systems exhibit unstable periodic orbits which usually appear embedded in chaotic attractors. Most of the research efforts were devoted to the control of chaotic behavior which can be found in many of these unstable orbits, a constant feature in low-dimensional dynamical systems. Such orbits can be controlled with small perturbation forces, which decrease as the system approaches the stabilized periodic orbit where those control forces vanish.

Both time-delayed techniques have been applied to control the attitude dynamics of spacecrafts. In this way, Fujii et al. [14] have used the TDAS control to stabilize a gravity-gradient satellite in the elliptic orbit. Kojima et al. [15,16] have also applied this method to both inert, and alive, electrodynamic tethered satellite systems in elliptic orbits in order to control their libration motions. Peláez and Lorenzini [17] used the TDAS technique to stabilize the attitude motion of electrodynamic tethers working in inclined circular orbits. In this work, the authors found that the TDAS scheme does not work well with electrodynamic tethers; though this control law delays the onset of instability, it does not stabilize the unstable periodic orbits for reasonable values of the control parameters. They suggested the use of the ETDAS method because it has been used with success in some cases where TDAS failed. In this sense, the ETDAS scheme has been applied to control the chaotic attitude motion of a magnetic spacecraft in the polar elliptic orbit [18]. Following the suggestion of Peláez and Lorenzini, the ETDAS method has also been applied, with success, by Iñarrea and Peláez [19] to stabilize the libration motions of electrodynamic tethers in inclined circular orbits.

The present paper goes one step forward in the application of these time-delayed feedback control methods, TDAS and ETDAS, to stabilize the unstable periodic libration motions of electrodynamic tethers in inclined elliptic orbits. The main idea is that stabilized periodic libration motions can be used as suitable starting points for the operation of the tether. The structure of the paper is organized as follows. In Section 2, we describe the tether model used in this study, the equations of the libration motion are also derived, and the basic periodic libration motions of the uncontrolled tether are described. Section 3 is devoted to explaining the particular application of both TDAS and ETDAS control methods to the tether dynamics in order to transform the unstable basic periodic libration motions into asymptotically stable ones. Section 4 is focused on the comparison of the effectiveness of the TDAS and ETDAS control techniques in the stabilization of the basic periodic libration motions. To this end, the control domains of both methods have been computed for different values of the system parameters, by means of the alternative stability proposed by Bleich and Socolar [13]. The conclusions of the study are presented in Section 5.

2. Attitude dynamics of the tether

2.1. Model description

The electrodynamic tether satellite considered in this paper is based on the so-called dumbbell model. In this model, the EDT is made up by two end point masses, m_1 and m_2, connected by a rigid conductive rod of mass m_t and length L. This tether model with a rigid conductive rod connecting both end subsatellites is a useful first approximation to study the libration motions of the tether. Nevertheless, the rigid rod approach does not take into account the elasticity and flexibility of a real tether. Therefore, this model is not able to account for other internal motions of the tether such as the longitudinal or transverse vibrations. However, the goal of this paper is not related to these other internal motions. On the other hand, any real tether in orbit hanging suspended in the local vertical direction is subject to tension along the tether due to its own weight. This tension increases linearly with the tether length. As a consequence of this, any real tether has a critical break length [20,21] beyond which the tether breaks. This break length is determined by the tether cross-section area, the orbital angular velocity, and also by the material properties, such as the density and Young's modulus. These references [20,21] include the values of those properties for some strong materials, as well as the corresponding break lengths, that can reach thousands of kilometres in the geostationary orbit.

The EDT system is orbiting around the Earth subject to the gravitational field that is modeled as due to a spherical central planet. On the other hand, the EDT satellite has plasma electric contactors at both ends, so that electric current can flow through the tether by closing the circuit via the ionospheric plasma. Therefore, the EDT is also affected by the geomagnetic field through the electrodynamic Lorentz force due to the tether electric current. In this paper, the Earth's magnetic field is modeled as a single magnetic dipole aligned with the Earth's axis of rotation, and the tether electric current is assumed to be constant and uniform along the tether length.

Besides the gravitational and electromagnetic forces, the EDT system is also subject to suitable control forces applied with the aim of stabilizing the attitude motion of the tether. We assume that the center of mass G of the EDT satellite follows a Keplerian elliptic orbit of eccentricity e and inclination i. We also suppose that the total mass m of the EDT system is large enough to neglect the effect of the electrodynamic drag on the tether orbit. Therefore, it is assumed that the trajectory followed by the satellite is a frozen Keplerian elliptic orbit not affected by the attitude motion of the EDT system.

In order to define the orbital position of the mass center of the tether, we use an inertial right-handed orthogonal geocentric reference frame $EXYZ$, with origin E located at the center of mass of the Earth. The XY plane is coincident with the equatorial plane, so that, the X-axis is in the direction of the first Point of Aries, and the Z-axis is aligned with the Earth's rotation axis. In this reference frame, the orientation of the satellite orbital plane is described by two orbital parameters, the right ascension of the ascending node Ω, and the inclination i, see Fig. 1. The position of the tether center of mass G in the orbit is defined by the orbital radius r, the argument of perigee $\tilde{\omega}$ and the true anomaly, ν.

On the other hand, in order to describe the attitude orientation of the EDT satellite, we use the right-handed orthogonal orbital frame $Gxyz$, with origin coincident with the tether center of mass. The direction of the x-axis is

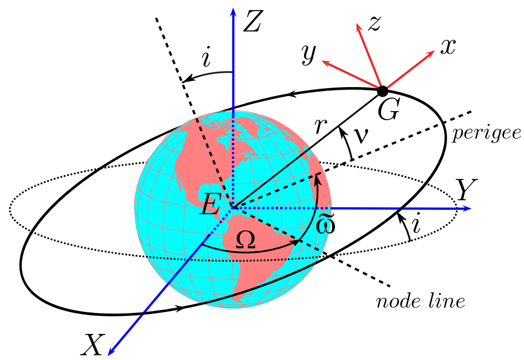

Fig. 1. Geocentric and orbital reference frames.

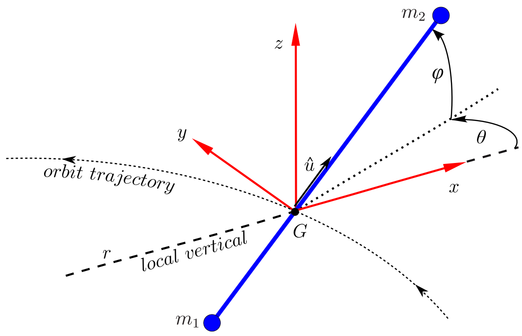

Fig. 2. Orbital reference frame and angular coordinates of the tether attitude.

along the local vertical pointing to zenith, and the z-axis is normal to the orbital plane. In this reference frame, the orientation of the tether is described by two angular coordinates: the out-of-plane angle φ, formed by the tether and the orbital plane ($-\pi/2 \le \varphi \le \pi/2$); and the in-plane angle θ, formed by the x-axis and the projection of the tether on the orbital plane ($-\pi \le \theta \le \pi$), see Fig. 2.

2.2. Equations of attitude motion

The kinetic energy T of the EDT spacecraft can be expressed as

$$T = \tfrac{1}{2} m\left(\dot{r}^2 + r^2\dot{\nu}^2\right) + \tfrac{1}{2}\, \boldsymbol{\omega}^T \mathbb{I} \boldsymbol{\omega}, \tag{1}$$

where $\boldsymbol{\omega}$ is the attitude angular velocity vector of the EDT about its center of mass G, $\mathbb{I}$ is the tensor of inertia of the system, and the dot means derivation with respect to time. The first term of Eq. (1) is the kinetic energy corresponding to the orbital motion of the satellite, whereas the second term is the kinetic energy of the libration motion.

Expressed in the body frame attached to the tether, the tensor of inertia $\mathbb{I}$ is a diagonal one, that is, $\mathbb{I} = \mathrm{diag}(0, I_y, I_z)$, being $I_y = I_z$ the moments of inertia of the system

$$I_y = I_z = \left[\frac{m_1 m_2 + \frac{1}{3} m_t (m_1 + m_2) + \frac{1}{12} m_t^2}{m}\right] L^2 = m^* L^2, \tag{2}$$

where m^* is the reduced mass of the EDT system. On the other hand, the components of the attitude angular velocity $\boldsymbol{\omega}$ expressed in the body frame are given by

$$\boldsymbol{\omega} = \{(\dot{\theta} + \dot{\nu})\ \sin\ \varphi,\ -\dot{\varphi},\ (\dot{\theta} + \dot{\nu})\ \cos\ \varphi\}. \tag{3}$$

Therefore, taking into account Eqs. (2) and (3), the kinetic energy of the EDT satellite results in

$$T = \tfrac{1}{2} m\left(\dot{r}^2 + r^2\dot{\nu}^2\right) + \tfrac{1}{2}\ m^* L^2 \left[\dot{\varphi}^2 + (\dot{\theta} + \dot{\nu})^2\ \cos^2 \varphi\right]. \tag{4}$$

Besides, the potential energy of the EDT system, due to the gravity gradient and to the finite dimension of the spacecraft, can be written in the following well-known binomial Taylor expansion [6]:

$$U = -\frac{\mu_g m}{r} + \frac{\mu_g m^* L^2}{2r^3}\left(1 - 3\ \cos^2\theta\ \cos^2\varphi\right), \tag{5}$$

where μ_g is the gravitational parameter of the Earth. Thus, the Lagrangian of the EDT satellite, excluding the electrodynamic and control forces, is given by

$$\begin{aligned}\mathcal{L} = T - U = {}& \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\nu}^2\right) + \frac{\mu_g m}{r} \\ & + \frac{m^* L^2}{2}\left[\dot{\varphi}^2 + (\dot{\theta} + \dot{\nu})^2\ \cos^2\varphi\right] \\ & - \frac{\mu_g m^* L^2}{2r^3}\left(1 - 3\ \cos^2\theta\ \cos^2\varphi\right).\end{aligned} \tag{6}$$

In the above expression, the first two terms correspond to the pure orbital motion of the spacecraft, and they do not affect the attitude dynamics of the EDT system.

The equations of the libration motion can be obtained via Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = \frac{\partial \mathcal{L}}{\partial q_i} + Q_{ed\ i} + Q_{c\ i},$$

where $Q_{ed\ i}$ and $Q_{c\ i}$ are the components of the generalized forces associated with the electrodynamic interaction and the control respectively, and q_i are the generalized coordinates, in this case, $q_i = \{\theta, \varphi\}$. Therefore, the equations of the libration motion can be written as

$$\begin{cases} m^* L^2 [(\ddot{\theta} + \ddot{\nu})\ \cos^2\varphi - 2\dot{\varphi}(\dot{\theta} + \dot{\nu})\ \sin\ \varphi\ \cos\ \varphi] \\ \quad = -\dfrac{3\ \mu_g\ m^* L^2}{r^3}\ \sin\ \theta\ \cos\ \theta\ \cos^2\varphi + Q_{ed\ \theta} + Q_{c\ \theta} \\ m^* L^2 \ddot{\varphi} = -m^* L^2 (\dot{\theta} + \dot{\nu})^2\ \sin\ \varphi\ \cos\ \varphi - \dfrac{3\mu_g m^* L^2}{r^3}\ \cos^2\theta\ \sin\ \varphi\ \cos\ \varphi \\ \qquad + Q_{ed\ \varphi} + Q_{c\ \varphi}. \end{cases} \tag{7}$$

The components of the generalized electrodynamic force, $(Q_{ed\ \theta}, Q_{ed\ \varphi})$, can be computed by means of D'Alembert's principle of virtual work [22]. In this way, the elemental components $(dQ_{ed\ \theta}, dQ_{ed\ \varphi})$ acting on an element ds of the tether rod are given by

$$dQ_{ed\ \theta} = d\boldsymbol{F}_{Lor} \cdot \frac{\partial \boldsymbol{R}_{te}}{\partial \theta}, \quad dQ_{ed\ \varphi} = d\boldsymbol{F}_{Lor} \cdot \frac{\partial \boldsymbol{R}_{te}}{\partial \varphi}, \tag{8}$$

where $\boldsymbol{R}_{te}$ is the position vector of the tether element ds with respect to the mass center E of the Earth, and $d\boldsymbol{F}_{Lor}$ is the elemental Lorentz force acting on the tether element. The expression of this elemental Lorentz force is

$$d\boldsymbol{F}_{Lor} = I(\hat{\boldsymbol{u}} \times \boldsymbol{B})\ ds, \tag{9}$$

where I is the electric current in the tether, $\hat{\boldsymbol{u}}$ is a unit vector in the direction of the tether, and $\boldsymbol{B}$, assumed to be constant along the tether, is the magnetic field of the Earth evaluated at the center of mass of the EDT satellite. The components of the vectors $\hat{\boldsymbol{u}}$ and $\boldsymbol{R}_{te}$, expressed in the orbital reference frame, can be written in the form

$$\hat{\boldsymbol{u}} = \{\cos\ \theta\ \cos\ \varphi,\ \sin\ \theta\ \cos\ \varphi,\ \sin\ \varphi\},$$

$$\boldsymbol{R}_{te} = \{r + s\ \cos\ \theta\ \cos\ \varphi, s\ \sin\ \theta\ \cos\ \varphi, s\ \sin\ \varphi\}, \tag{10}$$

where s is the position coordinate of the tether element ds measured along the tether with the origin at the center of mass G.

Taking into account Eqs. (9) and (10), the elemental components (8) result in the following expressions:

$$dQ_{ed\ \theta} = \cos\ \varphi[\ \sin\ \varphi(B_x\ \cos\ \theta + B_y\ \sin\ \theta) - B_z\ \cos\ \varphi] Is\ ds,$$

$$dQ_{ed\ \varphi} = (B_y \cos\theta - B_x \sin\theta) Is\, ds, \tag{11}$$

where (B_x, B_y, B_z) are the components of the geomagnetic field expressed in the orbital reference frame.

The components $(Q_{ed\ \theta}, Q_{ed\ \varphi})$ of the total generalized electrodynamic force acting on the tether rod can be calculated integrating the elemental components (11) along all the tether length. After this computation, the following expressions are obtained:

$$Q_{ed\ \theta} = \frac{L^2 I\Lambda}{2} \cos\varphi \left[\sin\varphi (B_x \cos\theta + B_y \sin\theta) - B_z \cos\varphi\right],$$

$$Q_{ed\ \varphi} = \frac{L^2 I\Lambda}{2} (B_y \cos\theta - B_x \sin\theta), \tag{12}$$

where $\Lambda = (m_1 - m_2)/m$. In this computation, we have assumed a uniform electric current I along the tether length.

We suppose that the terrestrial magnetic field $\boldsymbol{B}$ is generated by a perfect dipole located at the mass center E of the Earth and aligned with its rotation axis [23–25]. In this way, the components of the magnetic field are expressed in the orbital frame as

$$\begin{cases} B_x = -2\dfrac{\mu_m}{r^3} \sin i \sin(\nu + \tilde{\omega}), \\ B_y = \dfrac{\mu_m}{r^3} \sin i \cos(\nu + \tilde{\omega}), \\ B_z = \dfrac{\mu_m}{r^3} \cos i, \end{cases} \tag{13}$$

where μ_m is the geomagnetic dipole moment. Taking into account Eqs. (12) and (13), the equations of the libration motion (7) take the following form:

$$\begin{cases} m^* L^2 [(\ddot{\theta} + \ddot{\nu}) \cos^2\varphi - 2\dot{\varphi}(\dot{\theta} + \dot{\nu}) \sin\varphi \cos\varphi] \\ \quad = -\dfrac{3\mu_g m^* L^2}{r^3} \sin\theta \cos\theta \cos^2\varphi - \dfrac{\mu_m}{r^3} \dfrac{L^2 I\Lambda}{2} \cos\varphi \{\cos i \cos\varphi \\ \quad + \sin i \sin\varphi [2 \cos\theta \sin(\nu + \tilde{\omega}) - \sin\theta \cos(\nu + \tilde{\omega})]\} + Q_{c\ \theta}, \\ m^* L^2 \ddot{\varphi} = -m^* L^2 (\dot{\theta} + \dot{\nu})^2 \sin\varphi \cos\varphi - \dfrac{3\mu_g m^* L^2}{r^3} \cos^2\theta \sin\varphi \cos\varphi \\ \quad + \dfrac{\mu_m}{r^3} \dfrac{L^2 I\Lambda}{2} \sin i [\cos\theta \cos(\nu + \tilde{\omega}) + 2 \sin\theta \sin(\nu + \tilde{\omega})] + Q_{c\ \varphi}. \end{cases} \tag{14}$$

By means of a change of variable, the time t can be replaced by the true anomaly ν as the independent variable of the problem. This change of variable is performed making use of the orbit equation $\dot{\nu} = m^3 \mu_g^2 (1 + e \cos\nu)^2 / h^3$, where h is the orbital angular momentum of the EDT satellite. After some cumbersome algebra, the equations of the attitude motions become

$$\begin{cases} \theta'' = 2(\theta' + 1)\left[\dfrac{e \sin\nu}{1 + e \cos\nu} + \varphi' \tan\varphi\right] - \dfrac{3 \sin\theta \cos\theta}{1 + e \cos\nu} \\ \quad - \dfrac{\varepsilon}{1 + e \cos\nu} \{\cos i + \sin i \tan\varphi [2 \cos\theta \sin(\nu + \tilde{\omega}) \\ \quad - \sin\theta \cos(\nu + \tilde{\omega})]\} + \dfrac{Q_{c\ \theta}}{m^* L^2 \dot{\nu}^2} \cos^2\varphi \\ \varphi'' = \dfrac{2e \sin\nu}{1 + e \cos\nu} \varphi' - \left[(\theta' + 1)^2 + \dfrac{3 \cos^2\theta}{1 + e \cos\nu}\right] \sin\varphi \cos\varphi \\ \quad + \dfrac{\varepsilon}{1 + e \cos\nu} \sin i [\cos\theta \cos(\nu + \tilde{\omega}) + 2 \sin\theta \sin(\nu + \tilde{\omega})] \\ \quad + \dfrac{Q_{c\ \varphi}}{m^* L^2 \dot{\nu}^2}, \end{cases} \tag{15}$$

where the primes stand for the derivation with respect to the true anomaly ν, and the electrodynamic parameter ε is defined as

$$\varepsilon = \frac{\mu_m I\Lambda}{2\mu_g m^*}. \tag{16}$$

The parameter ε somehow gauges the ratio between the Lorentz and the gravitational forces acting upon the system, and it is also an indication of the strength of the electrodynamic interaction. The last term of both equations in (15) corresponds to the forces applied to control the libration dynamics of the EDT satellite. The expressions of the control forces considered in this study are defined in Section 3.

2.3. Uncontrolled tether: basic periodic motions

This section is focused on a certain set of unstable periodic attitude motions of the tether in the absence of control forces. This set of unstable periodic motions are the target of the time-delayed control schemes that are introduced in the next sections with the aim of stabilizing the libration motions of the tether. In the uncontrolled regime $(Q_{c\ \theta} = Q_{c\ \varphi} = 0)$, the equations of the attitude motion (15) involve four free parameters: the argument of perigee $\tilde{\omega}$, the orbital inclination i, the eccentricity e, and the electrodynamic parameter ε.

As is well known, for an inert tether with no current flowing through the rod $(\varepsilon = 0)$, in the circular orbit $(e = 0)$, the satellite exhibits four equilibrium attitude positions with respect to the orbital reference frame. All of them are located in the orbital plane $(\varphi = 0)$. Two of these equilibrium positions are unstable and the other two stable. In the unstable positions, the tether is aligned with the local horizontal $(\theta = \pm\pi/2)$, whereas in the stable ones it is oriented in the direction of the local vertical $(\theta = 0, \pi)$. Apart from these equilibrium positions, the dynamics of an inert tether in the circular orbit also exhibits periodic attitude oscillatory motions, in angles θ and φ, with periods equal to the periodicity of the true anomaly in the orbital motion, that is 2π, or multiples of it. Some of these periodic libration motions have been described in Refs. [4,5,8].

When an inert tether satellite $(\varepsilon = 0)$ is following an elliptic orbit $(e \neq 0)$ the local vertical orientations are no longer equilibrium positions. In this case, the spacecraft exhibits unstable periodic libration motion instead of such steady positions. Peláez and Andrés [8] have analyzed some of these periodic motions and their stability.

On the other hand, for an alive EDT satellite, that is, with current flowing through the tether $(\varepsilon \neq 0)$, even in an inclined circular orbit, the local vertical direction is not an equilibrium position either. In this case, the tether attitude dynamics also have periodic libration motions. However, all of them are unstable due to the dynamic instability generated by the electrodynamic interaction with the geomagnetic field [6]. Some of these periodic attitude motions, along with stability properties, have been studied in Ref. [7].

In this paper, we focus on the case of an alive EDT following an inclined elliptic orbit. We are interested in a

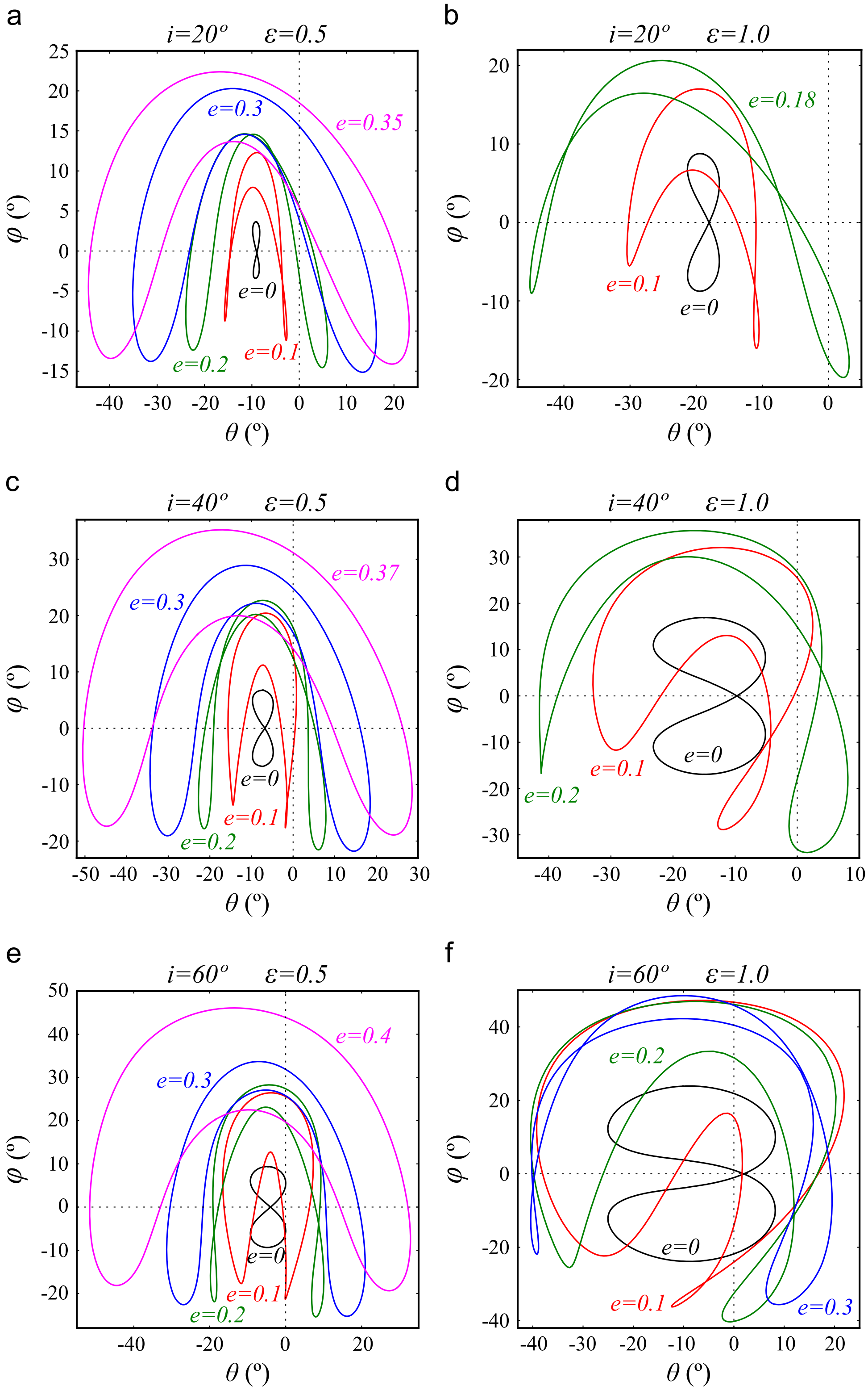

Fig. 3. Shape of the basic periodic libration motions in the (θ,φ) plane for different values of the system parameters i, ε and e.

certain group of periodic attitude motions, which we call basic periodic motions. Indeed, having fixed the value of $\tilde{\omega}$, for each set of values of the other free parameters, there is a particular periodic libration motion with the same period as the orbital motion. This 2π-periodic motion is directly related to the original stable equilibrium orientation of the tether attitude in the inert circular case ($\varepsilon = e = 0$). In fact, for each orbital inclination i, these particular periodic libration motions collapse to the stable equilibrium position along the local vertical when ε and e tend to zero. These special periodic attitude motions are the basic periodic motions considered in this paper. The dynamic stability of these basic periodic motions has already been analyzed by Peláez and Andrés [8]. In that study, the authors found that the effect of the argument of perigee $\tilde{\omega}$ in the stability of the basic periodic motions is very small compared to the effect of the eccentricity. That is the reason why we have fixed the value of $\tilde{\omega}$ and, in the present study, it will be set equal to zero.

In order to compute these basic (2π)-periodic motions, for each fixed orbital inclination i, we have made a numerical continuation of the solution of the stable equilibrium position along the local vertical of the inert circular case ($\varepsilon = e = 0$), using ε and e as the continuation parameters. This numerical continuation has been performed by means of the freely distributed software package AUTO2007 [26,27]. This software carries out the bifurcation analysis and continuation of solutions of systems of differential equations with respect to the parameters of the problem.

Fig. 3 shows the shape of these basic (2π)-periodic motions in the plane of the angular coordinates (θ,φ), for different values of the system parameters (i, ε, e). From this figure, it is clear that the amplitude of the in-plane and the out-of-plane oscillations grows with the orbital eccentricity in all cases. In this figure, only periodic motions appear for relatively small eccentricity values. The reason for this is the fact that, for each pair of fixed values of i and ε, the continuation of the family for increasing e ends for relatively small eccentricity values in a cyclic-fold bifurcation [28]. In this bifurcation, the family of basic periodic motions coalesces with another family of periodic motions that has its origin in a 2π-periodic oscillation motion of the inert circular case [8]. In this bifurcation, both families of periodic motions disappear for higher values of eccentricity.

Apart from these basic periodic motions, it is worth noting that the uncontrolled EDT in the inclined elliptic

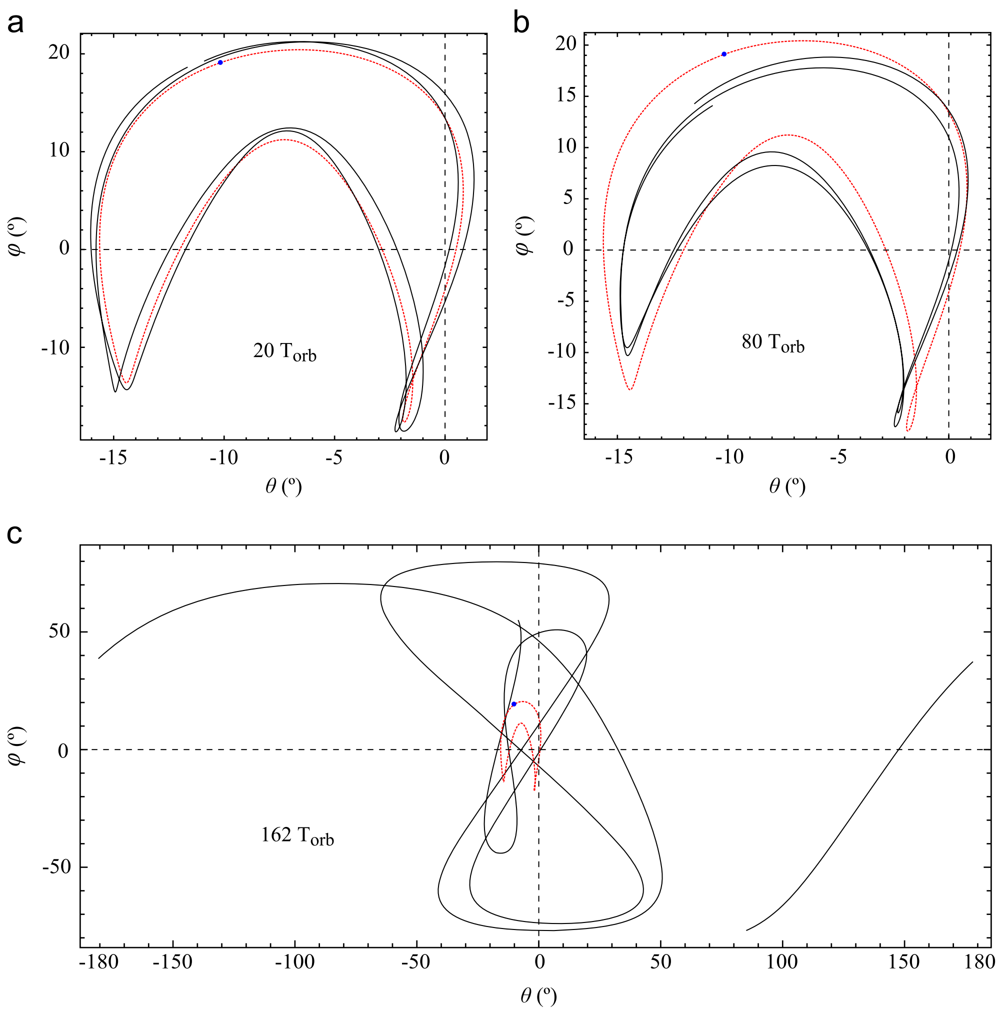

Fig. 4. Unstable behavior of a libration motion with initial conditions very close to the basic periodic solution (dashed line) corresponding to $i=40^{\circ}$, $\varepsilon=0.5$ and $e=0.1$ after different orbital periods. In each case, only the libration motion during the last two orbital periods is plotted.

orbit also has other 2π-periodic attitude motions. These other periodic motions emanate from the unstable equilibrium position along the local horizontal, or from periodic libration motions of the inert circular case. Moreover, the uncontrolled system exhibits other periodic motions whose periods are multiples of 2π. These secondary periodic libration motions arise from bifurcations suffered by the basic periodic motions during the continuation process by tuning parameters ε and e. Among the drawbacks of all these other periodic motions, their high instability and the large amplitude of their attitude oscillations may be mentioned.

When the system is not controlled, all of the basic 2π-periodic motions are also unstable for any values of the three free parameters i, ε and e [8].

Figs. 4 and 5 graphically represent two examples of the unstable character of the basic periodic motions of the uncontrolled tether. The dashed line represents the basic periodic solution, and the continuous line represents a libration motion starting from initial conditions very close to that periodic motion. The small dot is located at the initial conditions of the attitude motion. These uncontrolled motions have been computed by numerical integration of the equations of motion (15) with $Q_{c\,\theta}=Q_{c\,\varphi}=0$. Fig. 4 corresponds to the case $i=40^\circ$ and $\varepsilon=0.5$ for a small value of the orbital eccentricity $e=0.1$. The plot represents the attitude motion followed by the tether after 20, 80 and 162 orbital periods (only the corresponding last two periods are shown in the figure). From these graphs,

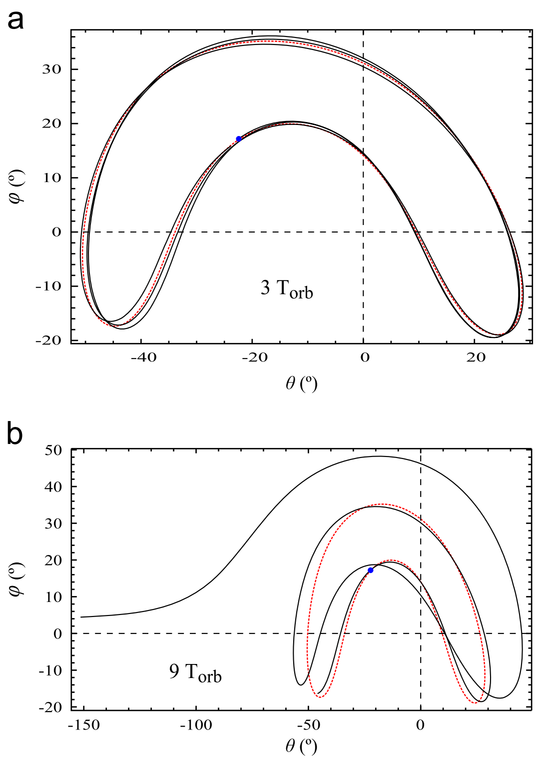

Fig. 5. Unstable behavior of a libration motion with initial conditions very close to the basic periodic solution (dashed line) in the case of $i=40^\circ$, $\varepsilon=0.5$ and $e=0.37$ after different orbital periods. In (b), only the libration motion during the last two orbital periods is plotted.

it is clear that although the motions start with initial conditions close to the periodic solution, after 162 orbital periods, the libration motion of the tether is very far away from the periodic trajectory. In fact, the motion has undergone a transition from oscillation to rotation. Fig. 5 shows another example of the instability of the basic periodic solutions for the same case $i=40^\circ$ and $\varepsilon=0.5$, but for a larger value of the orbital eccentricity $e=0.37$. In this case, the corresponding basic periodic motion is much more unstable. As can be seen in Fig. 5b, after only 9 orbital periods, the libration motion of the tether is very far away from the periodic motion, undergoing the transition from libration to rotation.

These two examples graphically demonstrate the fact that the instability of the basic periodic attitude motions increases with the eccentricity e. Peláez and Andrés [8] have done an extensive analysis of the eigenvalues of the monodromy matrix of the periodic motions of the uncontrolled electrodynamic tether. They studied the dependence of the eigenvalues with the free parameters of the system. They also showed that the instability of the basic periodic motions increases with the eccentricity e.

3. Libration control with two time-delay feedback methods

This section is devoted to the application of two time-delayed feedback control methods to the system dynamics in order to transform the unstable character of the basic periodic attitude motions of the uncontrolled tether into asymptotically stable periodic ones. The first one of these schemes is the so-called time-delay autosynchronization (TDAS) method [10]. The second control technique is a natural extension of the first one and so, it is called the extended time-delay autosynchronization (ETDAS) method [11,12]. These two methods have two important advantages: they do not require fast switching or sampling, nor do they need a reference signal corresponding to the desired regular motion. They only require the knowledge of the period of the target periodic orbit.

With respect to the TDAS control scheme, Fig. 6 shows the basic block diagram that describe the method. The control variable y of the system is delayed at the output by some amount of time τ, and then it is re-introduced into the system through the feedback control signal $F(t)=k[y(t-\tau)-y(t)]$. When considering periodic motions, the delay time τ coincides with the period of the orbit. This control perturbation can be adjusted through the parameter k in order to achieve the stabilization of the desired periodic orbit, that is, k is a free parameter of the problem. It is important to note that, when the controlled system

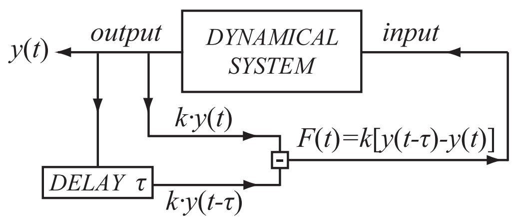

Fig. 6. Block diagram of the TDAS control method.

follows a periodic orbit of period τ, the control signal $F(t)$ vanishes for any value of k, because in that case $y(t-\tau)=y(t)$.

The application of the control signal $F(t)$ to the system implies the existence of additional control forces acting on the tether satellite. These additional forces have to be taken explicitly into account in the equations of the attitude motion as the control terms. The TDAS control technique has been already used to convert unstable periodic motions into stable ones in the case of EDT in inclined circular orbits [17]. Following that study, in order to apply the TDAS method in this case, we assume that the generalized control forces $(Q_{c\,\theta}, Q_{c\,\varphi})$ of Eqs. (15) have the following explicit form:

$$Q_{c\,\theta} = m^* L^2 \dot{\nu}^2 \cos^2\varphi \, F_\theta(\nu), \quad Q_{c\,\varphi} = m^* L^2 \dot{\nu}^2 F_\varphi(\nu), \tag{17}$$

where the two control signals $F_\theta(\nu)$ and $F_\varphi(\nu)$, according to the TDA technique, are given by

$$F_\theta(\nu) = k_\theta[\dot\theta(\nu-\tau) - \dot\theta(\nu)], \quad F_\varphi(\nu) = k_\varphi[\dot\varphi(\nu-\tau) - \dot\varphi(\nu)]. \tag{18}$$

This means that the control variables we choose are the angular velocities $\dot\theta$ and $\dot\varphi$. The delay time τ must be the period of the unstable periodic motions we want to stabilize. As in this problem time has been replaced by the true anomaly ν as the independent variable, the delay time must be $\tau=2\pi$. In this way, there are two new parameters, k_θ and k_φ, in the added control terms to achieve the stabilization of the basic periodic librational motions of the tether. Thus, taking into account Eqs. (17) and (18), the equations of attitude motion under the application of the TDAS control method can be written as

$$\left\{\begin{array}{lcl}
\theta'' & = & 2(\theta'+1)\left[\dfrac{e\sin\nu}{1+e\cos\nu} + \varphi'\tan\varphi\right] - \dfrac{3\sin\theta\cos\theta}{1+e\cos\nu} \\
 & & -\dfrac{\varepsilon}{1+e\cos\nu}\{\cos i + \sin i \tan\varphi[2\cos\theta\sin(\nu+\tilde\omega) \\
 & & -\sin\theta\cos(\nu+\tilde\omega)]\} + F_\theta(\nu), \\
\varphi'' & = & \dfrac{2e\sin\nu}{1+e\cos\nu}\varphi' - \left[(\theta'+1)^2 + \dfrac{3\cos^2\theta}{1+e\cos\nu}\right]\sin\varphi\cos\varphi \\
 & & +\dfrac{\varepsilon}{1+e\cos\nu}\sin i[\cos\theta\cos(\nu+\tilde\omega) + 2\sin\theta\sin(\nu+\tilde\omega)] \\
 & & +F_\varphi(\nu).
\end{array}\right. \tag{19}$$

It should be noted that, when the controlled tether follows a 2π-periodic orbit, both control signals F_θ and F_φ vanish. Indeed, any 2π–periodic motion of the uncontrolled tether is also a 2π-periodic orbit of the controlled one. Unfortunately, Peláez and Lorenzini showed in their study [17] that the TDAS control method fails to stabilize the basic periodic motions of the tether. Thus, this control technique is not able to convert the unstable periodic motions of the uncontrolled tether into asymptotically stable ones. In that study, the authors proposed the use of the ETDAS control scheme to stabilize the unstable periodic motions of the EDT.

The basic block diagram of the ETDAS control method is shown in Fig. 7. In this technique, the control variable y is progressively delayed at the output by multiples of some amount of time τ. All of these delayed control values $y(t-j\tau)$ are then re-introduced into the system through the

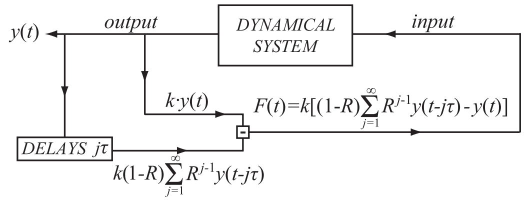

Fig. 7. Block diagram of the ETDAS control method.

feedback control signal given by

$$F(t) = k\left[(1-R)\sum_{j=1}^{\infty} R^{j-1} y(t-j\tau) - y(t)\right].$$

This control signal has two adjustable parameters, the feedback gain k and the memory parameter $0 \le R < 1$.

As in the TDAS, when the ETDAS technique is applied to a periodic motion, the delay time τ must coincide with the period of the motion. However, the ETDAS method uses information about many previous states of the system in order to stabilize the periodic orbit with period τ. It is worth emphasizing that, when the system controlled by ETDAS follows a τ-periodic orbit, the control signal $F(t)$ also vanishes for any values of the control parameters R and k, as it happens in the TDAS control scheme. Indeed, in that case $y(t-j\tau)=y(t)$ for all j, and then the identity $1/(1-R) = \sum_{k=0}^{\infty} R^k$ leads to the vanishing of the control signal $F(t)$. Note also that, in the limit $R\to 0$, the ETDAS method coincides with TDAS. That is, the TDAS control technique is a limit case of ETDAS.

The ETDA control scheme has been already used with success to convert unstable periodic motions into stable ones in the case of electrodynamic tethers in inclined circular orbits [19]. In order to apply the ETDAS method in this case, we follow the same line of that study, and we assume that the generalized control forces $(Q_{c\,\theta}, Q_{c\,\varphi})$ of Eqs. (15) have the same explicit form of (17). Nevertheless, in this case, and according to the ETDAS technique, the two control signals, $F_\theta(\nu)$ and $F_\varphi(\nu)$, are given by

$$F_\theta(\nu) = k_\theta\left[(1-R_\theta)\sum_{j=1}^{\infty} R_\theta^{j-1}\dot\theta(\nu-j\tau) - \dot\theta(\nu)\right],$$

$$F_\varphi(\nu) = k_\varphi\left[(1-R_\varphi)\sum_{j=1}^{\infty} R_\varphi^{j-1}\dot\varphi(\nu-j\tau) - \dot\varphi(\nu)\right]. \tag{20}$$

Therefore, in both methods we choose the same control variables, that is, the angular velocities $\dot\theta$ and $\dot\varphi$. In this case, we have four different control parameters k_θ, k_φ, and R_θ, R_φ, with $0 \le R_i < 1$ to achieve the stabilization of the basic periodic libration motions. As it happens in the TDAS control scheme, when the tether controlled by ETDAS follows a 2π-periodic motion, the control signals F_θ and F_φ vanish. Therefore, any 2π-periodic motion of the uncontrolled tether is also a 2π-periodic orbit of the tether controlled by the ETDAS method.

Both feedback control techniques, TDAS and ETDAS, share an attractive feature. When any of these methods is successfully applied to stabilize an initially unstable periodic libration motion, it is transformed into an asymptotically stable one. Therefore, any motion of the controlled

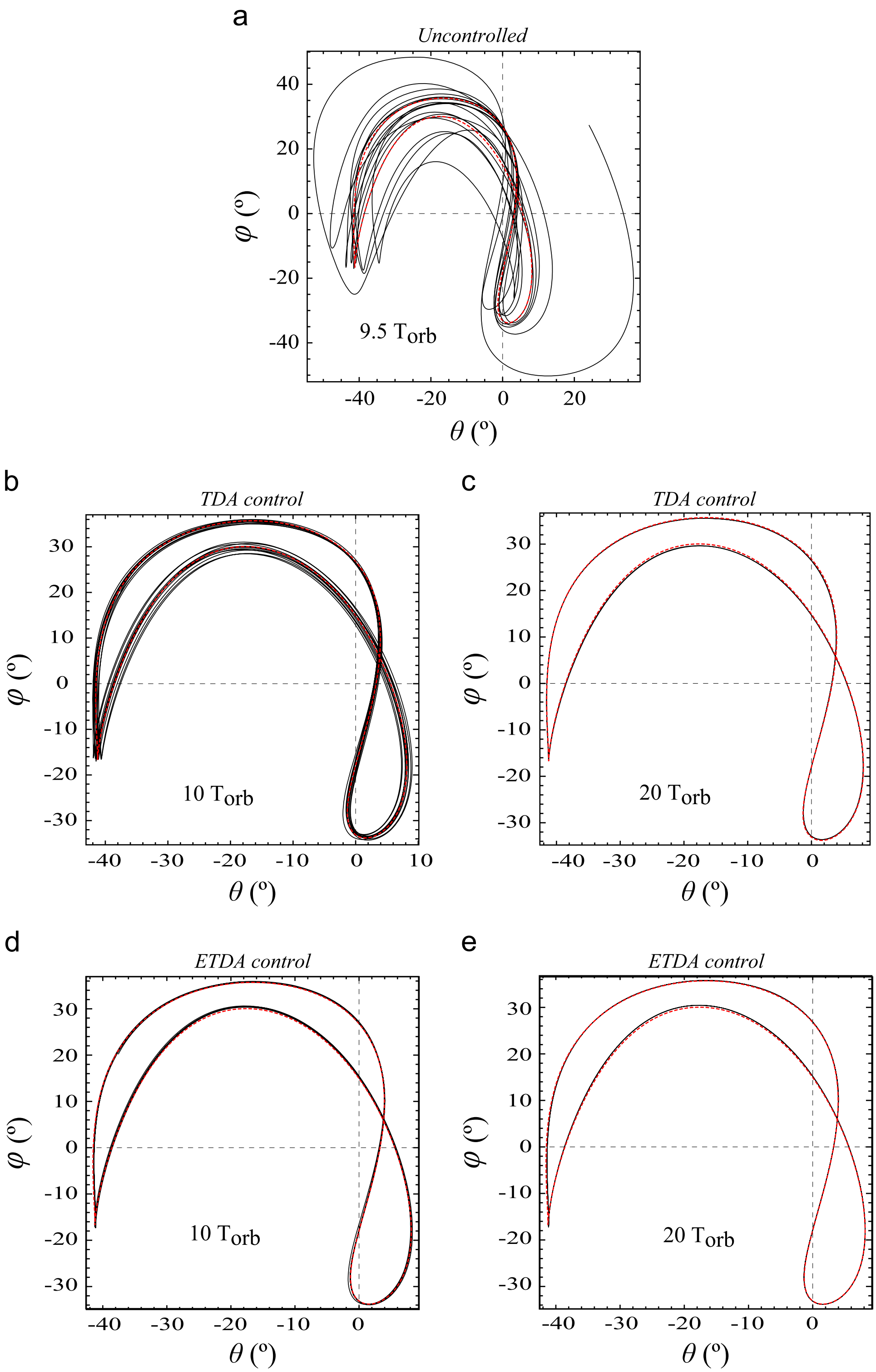

Fig. 8. Example of the successful application of both control methods for the case $i=40^{\circ}$, $\varepsilon=1.0$ and $e=0.2$. (a) Uncontrolled libration motion. (b and c) TDAS controlled motion. (d and e) ETDAS controlled motion. The control parameters used are $k_\theta=k_\varphi=R_\theta=R_\varphi=0.5$. The dashed line stands for the corresponding basic periodic motion. In (c) and (e) only the libration during the three last orbital periods is represented. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

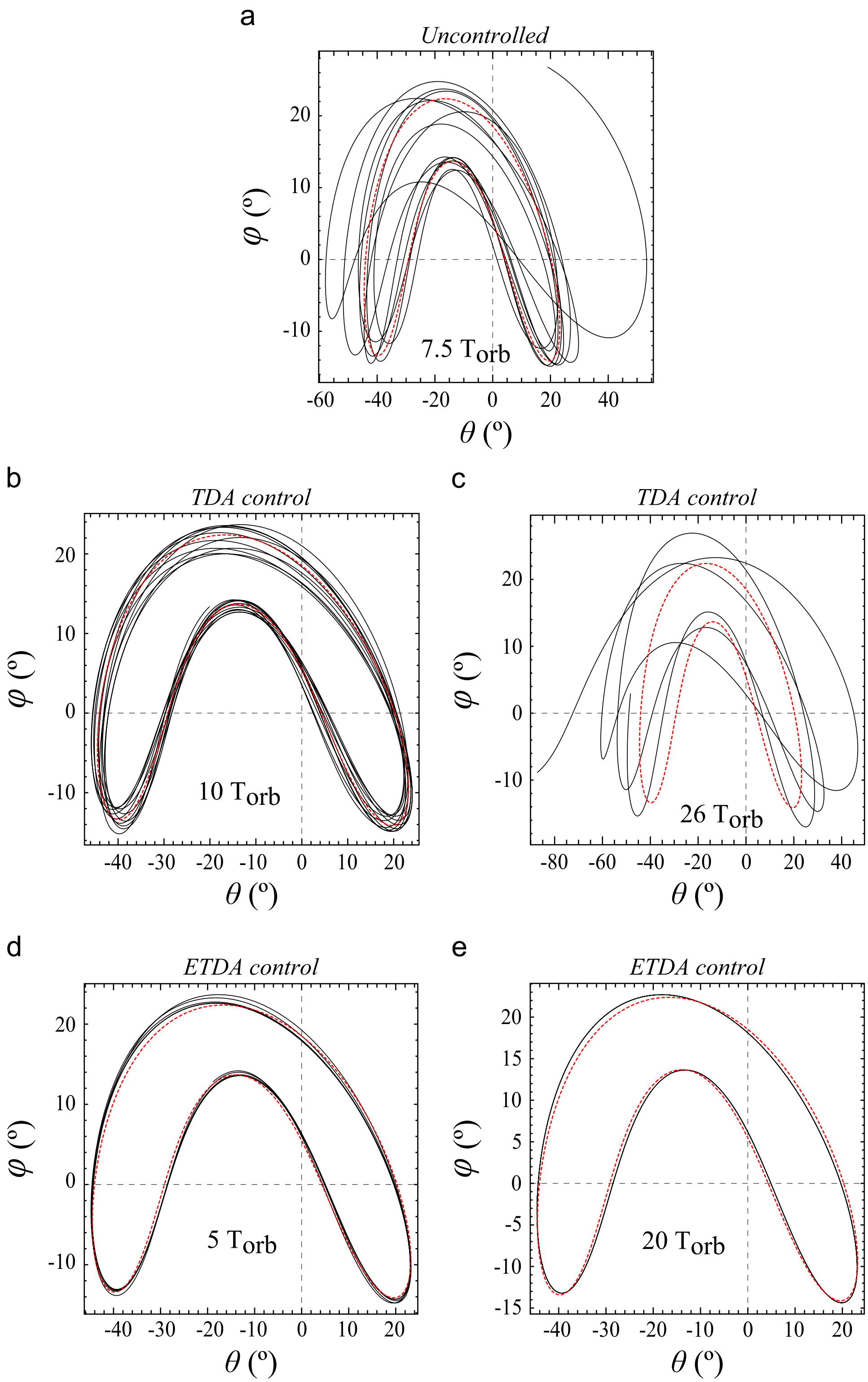

Fig. 9. Example of application of both control methods for case, $i = 20^{\circ}$, $\varepsilon = 0.5$ and $e = 0.35$. In this case the TDAS control fails in the stabilization of the basic periodic motion, but the ETDAS control successes. The control parameters used here are $k_{\theta} = 0.8$, $k_{\varphi} = 0.2$ and $R_{\theta} = R_{\varphi} = 0.5$. The dashed line stands for the corresponding basic periodic motion. In (c) and (e) only the libration during the three last orbital periods is represented.

system starting in the attraction basin of that stabilized periodic libration would approach it over time and, after a while, the control terms $(Q_{c\,\theta}, Q_{c\,\varphi})$ become very small because they would tend to zero when $\nu \to \infty$. Thus, if from the very beginning the tether is operated close to the basic periodic motion, it can be controlled with small controlling forces. This is a very interesting feature of both control methods. In this way, the growth of the oscillation amplitude of angles θ and φ, due to the instability of the uncontrolled tether, could be removed by the control forces.

Figs. 8 and 9 show two examples of application of both control methods to the tether libration dynamics. These examples have been computed by the numerical integration of equations of motion (15), controlled by each one of both techniques. The example shown in Fig. 8 corresponds to the case $i=40^\circ$, $\varepsilon=1.0$ and $e=0.2$. The red dashed line represents the basic periodic attitude motion corresponding to those values of the system parameters. The continuous black line stands for a libration motion with initial conditions close to the basic periodic motion. Fig. 8a shows the attitude motion of the uncontrolled tether during the first 9.5 orbital periods. As it can be seen in this figure, the uncontrolled motion goes far away from the basic periodic motion, tending to develop a transition from libration to rotation. Fig. 8b and c represents the attitude motion controlled by the TDAS method with the same initial conditions after 10 and 20 orbital periods respectively. For the sake of clarity, Fig. 8c only shows the libration motion during the three last orbital periods. Fig. 8d and e plots the tether attitude motion controlled by the ETDAS scheme with the same initial conditions after 10 and 20 orbital periods respectively. The values of the control parameters used in these tests are the same in both methods $k_\theta = k_\varphi = R_\theta = R_\varphi = 0.5$. As it can be seen in Fig. 8, both control techniques success in the stabilization of the basic periodic motion for those values of the control parameters. After 20 orbital periods, the tether libration practically coincides with the basic periodic motion. Therefore, both control methods are able to remove the unstable character of the basic periodic motion, transforming it into an asymptotically stable one.

Fig. 9 shows another example of application of both control schemes for the case $i=20^\circ$, $\varepsilon=0.5$ and $e=0.35$. Fig. 9a plots the uncontrolled attitude motion during the first 7.5 orbital periods for initial conditions close to the corresponding basic periodic motion. Fig. 9b and c and 9d and e represent the evolution of the libration motion controlled by the TDAS and ETDAS methods respectively. In this example, the values of the control parameters have been taken as $k_\theta = 0.8$, $k_\varphi = 0.2$ and $R_\theta = R_\varphi = 0.5$. Unlike the first example, Fig. 9 shows that, in this case, the ETDAS technique is more effective than the TDAS in stabilizing the basic periodic motion. Indeed, in Fig 9c it is clear that after 26 orbital periods the tether attitude motion controlled by TDAS has moved far away from the basic periodic motion, tending to perform rotation. On the other hand, Fig. 9e shows that after 20 orbital periods, the libration motion controlled by ETDAS tends to coincide with the basic periodic motion. Thus, in this case, and for these values of the control parameters, the ETDAS control technique successes in the stabilization of the basic periodic orbit, whereas the TDAS fails. The results of these examples of application of both control methods lead us to perform a comparative study of the effectiveness of both control schemes.

4. Effectiveness of the libration control

This section is focused on the comparison of the effectiveness of the TDAS and ETDAS control methods in the stabilization of the basic periodic libration motions of the tether. To this end, we have carried out a stability analysis of the basic periodic motions controlled by means of both methods. This stability analysis has been performed using the technique proposed by Bleich and Socolar [13]. This technique has been already used to compute the control domains in the case of electrodynamic tethers in inclined circular orbits controlled by the EDTAS method [19]. For the sake of the completeness of the paper, this stability analysis method is briefly described in the following paragraphs.

Let us consider an uncontrolled dynamical system with equations of motion

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, t)$$

where $\mathbf{y}$ is the n-dimensional vector which describes the dynamical state of the system. An unstable periodic orbit (UPO), $\mathbf{y}_p(t)$, of the uncontrolled system with period τ is known, and a nearby orbit $\mathbf{y}(t)$ is also considered. The goal is to control the system so that the UPO becomes a stable periodic orbit in such a way that the nearby orbit $\mathbf{y}(t)$ tends asymptotically to the periodic orbit $\mathbf{y}_p(t)$. This implies that the difference between both orbits $\mathbf{x}(t) = \mathbf{y}(t) - \mathbf{y}_p(t)$ must satisfy the condition: $\lim_{t\to\infty}\mathbf{x}(t) = 0$. To achieve this objective, the dynamical system is modified by the addition of a suitable control signal to the equations of motion. For the two feedback control methods considered in this paper, the corresponding controlled equations of motion can be written as

$$\begin{aligned}
\dot{\mathbf{y}}(t) &= \mathbf{f}(\mathbf{y}, t) + k\,\mathbf{M}[\mathbf{y}(t-j\tau) - \mathbf{y}(t)] && \text{TDAS,} \\
\dot{\mathbf{y}}(t) &= \mathbf{f}(\mathbf{y}, t) + k\mathbf{M}\left[(1-R)\sum_{j=1}^{\infty} R^{j-1}\mathbf{y}(t-j\tau) - \mathbf{y}(t)\right] && \text{ETDAS,}
\end{aligned} \tag{21}$$

where $\mathbf{M}$ is an $n \times n$ matrix which contains the information about the specific way the feedback control signal is applied to the system. Note that the periodic orbit $\mathbf{y}_p(t)$ is also a periodic solution of the controlled system (21). To study the stability properties of the UPO $\mathbf{y}_p(t)$ in the new controlled system, the time derivative of the deviation $\mathbf{x}(t)$ is written to first order as

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{J}(t)\,\mathbf{x}(t) + k\,\mathbf{M}[\mathbf{x}(t-j\tau) - \mathbf{x}(t)] && \text{TDAS,} \\
\dot{\mathbf{x}}(t) &= \mathbf{J}(t)\,\mathbf{x}(t) + k\,\mathbf{M}\left[(1-R)\sum_{j=1}^{\infty} R^{j-1}\,\mathbf{x}(t-j\tau) - \mathbf{x}(t)\right] && \text{ETDAS,}
\end{aligned} \tag{22}$$

where $\mathbf{J}(t)$ is the Jacobian matrix of the uncontrolled dynamical system.

The goal of the control method is to transform the UPO into an asymptotically stable orbit. Therefore, a suitable form for the solutions $\mathbf{x}(t)$ of (22) is $\mathbf{x}(t) = \mathbf{p}(t)e^{\lambda t/\tau}$, where $\mathbf{p}(t)$ is a τ-periodic function, $\mathbf{p}(t+\tau) = \mathbf{p}(t)$, and λ is a complex number with $\Re(\lambda) < 0$. Inserting this solution into (22)

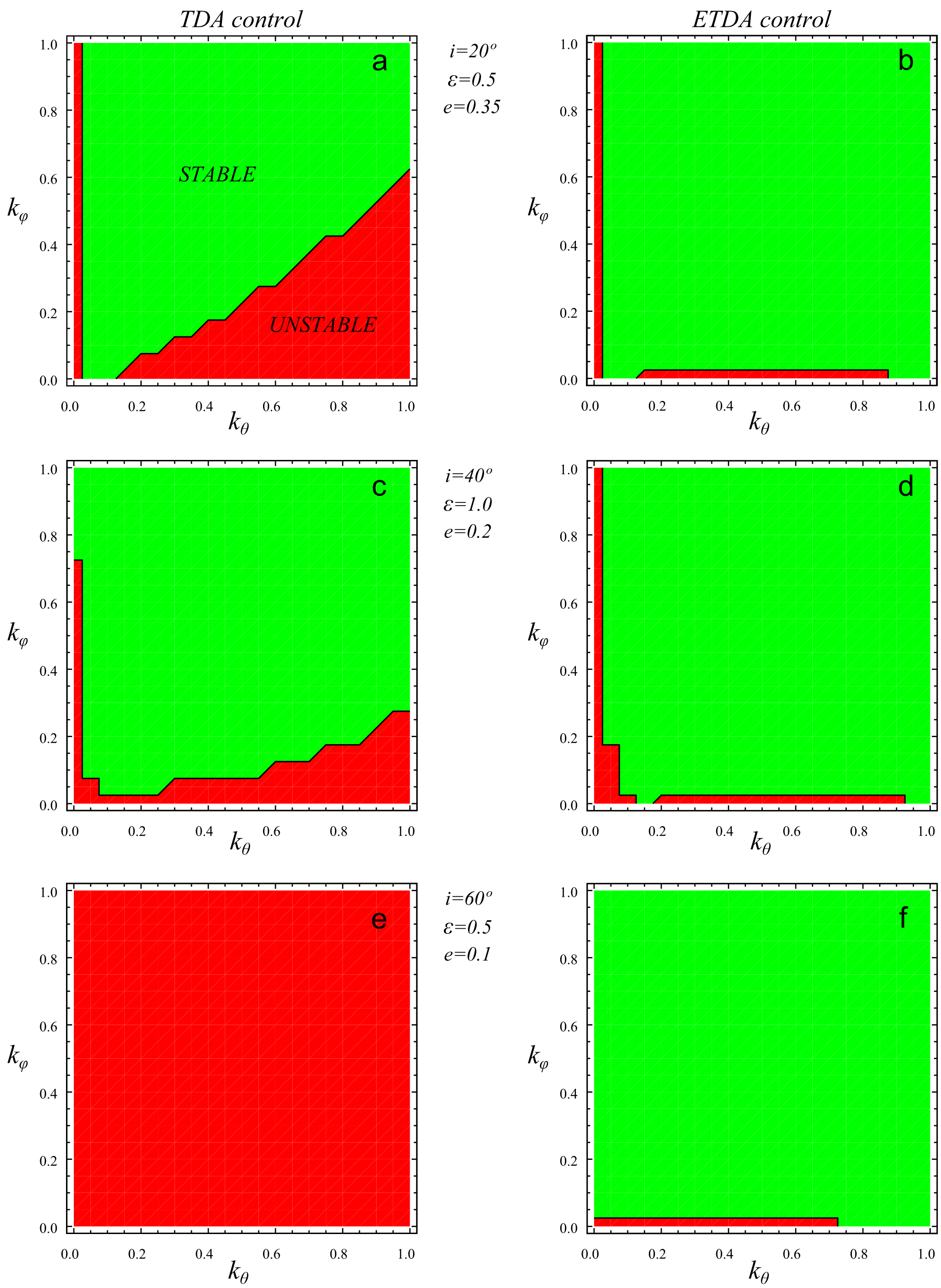

Fig. 10. Control domains of the TDAS (left) and ETDAS (right) methods in the parametric plane (k_θ, k_φ) for different values of the system parameters i, ε and e. In the ETDAS method the control parameters are taken as $R_\theta = R_\varphi = 0.5$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

gives

$$\dot{\mathbf{p}}(t) = \left[\mathbf{J}(t) - \frac{\lambda}{\tau}\,\mathbf{I} + k\,\mathbf{M}\,e^{-j\lambda} - k\,\mathbf{M}\right]\mathbf{p}(t) \qquad \text{TDAS,}$$

$$\dot{\mathbf{p}}(t) = \left[\mathbf{J}(t) - \frac{\lambda}{\tau}\,\mathbf{I} + k\,\mathbf{M}(1-R)\sum_{j=1}^{\infty} R^{j-1}\,e^{-j\lambda} - k\,\mathbf{M}\right]\mathbf{p}(t) \qquad \text{ETDAS,}$$

where $\mathbf{I}$ is the identity matrix. The solution of these differential equations for a given initial condition can be written as

$$\mathbf{p}(t) = e^{-\lambda t/\tau}\,\mathbf{U}(t)\,\mathbf{p}(0), \tag{23}$$

where $\mathbf{U}(t)$ is a matrix, which is the solution of the following problem:

$$\begin{aligned}\dot{\mathbf{U}}(t) &= [\mathbf{J}(t)+k\,\mathbf{M}\,(e^{-\lambda}-1)]\,\mathbf{U}(t) \quad \text{TDAS},\\ \dot{\mathbf{U}}(t) &= \left[\mathbf{J}(t)+k\,\mathbf{M}\,\frac{e^{-\lambda}-1}{1-R\,e^{-\lambda}}\right]\mathbf{U}(t) \quad \text{ETDAS},\end{aligned} \tag{24}$$

with the initial condition $\mathbf{U}(0)=\mathbf{I}$. Since $\mathbf{p}(t)$ is a τ-periodic function, the periodicity condition $\mathbf{p}(0)=\mathbf{p}(\tau)$ can be written as

$$[e^{-\lambda}\,\mathbf{U}(\tau)-\mathbf{I}\,]\,\mathbf{p}(0)=0,$$

where the relation (23) has been taken into account. As a consequence, the following determinant vanishes:

$$g(\mu^{-1})\equiv \det[\,\mu^{-1}\,\mathbf{U}(\tau)-\mathbf{I}\,]=0, \tag{25}$$

where $\mu\equiv e^{\lambda}$ is the Floquet multiplier.

The control method will be effective if, for any solution of Eq. (24), the corresponding deviation $\mathbf{x}(t)=\mathbf{p}(t)e^{\lambda t/\tau}$ goes asymptotically to zero, which means that all solutions of Eq. (25) must satisfy $\Re(\lambda)<0$. Therefore, the asymptotic stability of the periodic orbit $\mathbf{y}_p$ in the controlled system requires that all zeros of $g(\mu^{-1})$ lie outside the unit circle as $\|\mu^{-1}\|>1 \Leftrightarrow \Re(\lambda)<0$. For $R<1$, the determinant $g(\mu^{-1})$ has no poles inside the unit circle. Thus, by virtue of the Cauchy's argument principle [29], the number of roots of $g(\mu^{-1})$ inside the unit circle is equal to the number of times the path traced by $g(\mu^{-1})$ winds around the origin as μ^{-1} runs completely over the unit circle. Hence, the periodic orbit in the controlled system is stable if, and only if, this number of encirclements vanishes.

Therefore, the determination of the stability character of a particular periodic orbit in the controlled dynamical system by means of this technique involves the following consecutive tasks. First of all, the computation of the initial conditions and period τ of that particular UPO in the uncontrolled system. Second, the calculation of the matrix $\mathbf{U}(\tau)$ by the integration of Eq. (24) between 0 and τ. Third, the computation of the determinant $g(\mu^{-1})$ for a sequence of sufficiently closely spaced values of μ over the unit circle. And last, the determination of the number of encirclements of the origin corresponding to the path traced by $g(\mu^{-1})$.

Although this method of stability analysis involves some cumbersome calculations, it has several important advantages. The method avoids the delicate matter of the integration of the equations of motion of the controlled system with time-delay. This alternative method of stability analysis only requires the integration of the equations of motion without the time-delay control terms over only one period of the corresponding periodic orbit. Basically, this method reduces to the calculation of the number of encirclements of the origin of a curve in the complex plane.

By means of this technique of stability analysis, we have calculated the stability domains of the controlled tether for both TDAS and ETDAS control methods. The stability domains have been computed for different values of the orbital parameters of the system i and e, and the electrodynamic parameter ε. As the two control schemes applied in this paper share the control parameters k_θ and k_φ, the stability domains have been calculated in the parametric plane (k_θ, k_φ) with $0\le k_i\le 1$, with the aim of comparing the effectiveness of both control techniques. For the sake of simplicity, this comparative study of the control domains has been limited to the case in which the EDTAS control parameters $R_\theta=R_\varphi=0.5$ as a medium value in the range of these parameters.

Fig. 10 shows three representative examples of domains of control for both TDAS and ETDAS methods in the parametric plane (k_θ, k_φ), calculated by means of this alternative stability analysis for three different sets of values of the parameters i, ε and e. Green regions stand for the domains where the control method succeeds in stabilizing the periodic motion, whereas red regions stand for the domains where the control method fails. These domains of control have been calculated upon a two-dimensional grid of values of the control parameters (k_θ, k_φ) with steps of 0.05. For each one of the control methods, we have carried out the stability analysis of the controlled periodic motion corresponding to each pair of the control parameters in order to know its dynamical character.

The control domains shown in Fig. 10a and b correspond to the examples of controlled libration motions shown in Fig. 9, whereas the control domains shown in Fig. 10c and d correspond to the examples of controlled motions of Fig. 8. As can be seen in these figures, the dynamical behaviors of those controlled motions are in agreement with the corresponding control domains. Indeed, the values of the control parameters (k_θ, k_φ) used in the successful control cases lie in the stable regions (green), whereas the values used in the unsuccessful case lie in a unstable region (red). We can observe in Fig. 10 that, for those three cases of orbital and electrodynamic conditions, the ETDAS control technique seems to be more effective than the TDAS one in the stabilization of the basic periodic libration motions of the tether.

In order to get a quantitative comparison of the effectiveness of both control methods, the success rate of each

Table 1
Success rates in percentage of both feedback control methods for different values of the orbital parameters i and e, and the electrodynamic parameter ε.

i (°)	ε	e	% success TDAS	% success ETDAS
20	0.5	0.1	0	95.2
		0.2	93.2	93.2
		0.35	69.2	91.8
	1.0	0.1	85.0	95.2
		0.13	91.8	95.2
		0.17	89.8	91.1
40	0.5	0.1	0	94.3
		0.2	91.1	93.7
		0.3	91.8	90.5
	1.0	0.1	0	61.0
		0.15	79.4	87.5
		0.2	83.9	90.7
60	0.5	0.1	0	96.6
		0.2	89.6	94.1
		0.3	92.7	95.2
	1.0	0.1	0	25.8
		0.2	63.0	70.3
		0.3	29.0	29.9

technique has been computed for several values of the orbital parameters i and e, as well as the electrodynamic parameter ε. We have defined this success rate in the following way. For each set of values of the system parameters (i, ε, e), the corresponding control domain has been calculated using a number N_T of values of the control parameters (k_θ, k_φ) evenly distributed. Let N_s be the number of pair values (k_θ, k_φ) for which the control method successes. Therefore, the success rate of the control scheme for those values of the system parameters will be the ratio N_s/N_T. Table 1 shows the values of the success rate of both control methods calculated for several values of the system parameters. We have considered three different orbital inclinations, and two values of the electrodynamic parameter. For each pair of values (i,ε), we have taken into account three different values of the eccentricity.

From these results concerning the success rates, some remarks can be pointed out. First of all, taking into account all cases as a whole, the ETDAS control method is slightly more effective than the TDAS method in the stabilization of the basic periodic libration motions. It is worth noticing that in most of the studied cases where $e=0.1$, the TDAS method fails completely with zero values of the success rate. This fact seems to be in agreement with the failure of the TDAS method in the stabilization of periodic motion in the case of tethers in inclined circular orbits that has been studied by Peláez and Lorenzini [17]. Indeed, Iñarrea and Peláez showed that the ETDAS method is more effective in stabilizing the periodic libration motions in the case of inclined circular orbits [19].

On the other hand, in certain cases such as $i=60°$, $\varepsilon=1.0$ and $e=0.3$, the success rates of the control methods take quite small values in comparison with the rest of the cases. For these values of the system parameters, the corresponding uncontrolled basic periodic motions are highly unstable with large values of the modulus of the eigenvalues of the associated monodromy matrix. The existence of such highly unstable periodic libration motions in inclined orbits for certain ranges of the system parameters has already been pointed out by Peláez and Andrés [8]. Therefore, it is quite reasonable that the success rates of both control methods are quite lower in such unstable periodic motions.

According to the results shown in Table 1, the TDAS control method is slightly less effective than the ETDAS one. However, the greater simplicity of the TDAS method makes it preferable to the ETDAS one in order to stabilize the periodic libration motions of the tether. The more complex ETDAS control method should be used only on those cases of small orbital eccentricity, in which the TDAS method fails completely.

With respect to the practical application of both control techniques, in general, the experimental implementation of a feedback control method involves three steps [30]: the measurement of the actual state of the system, the generation of the suitable feedback signal, and finally the adjusting of some available actuator of the system in order to modify its evolution. The TDAS and ETDAS schemes proposed in this paper are based on the libration angular velocities $(\dot{\theta}, \dot{\varphi})$ of the tether. Therefore, the practical implementation of both control methods should include the following devices and actions performed in loop. First, rate-gyro sensors located at the tether end subsatellites would measure the libration angular velocities. A periodic sample of these measurements would be stored in the memory of the control CPU. This CPU would perform the algorithm of the control technique taking into account both the actual and the previous values of the libration angular velocities with delays of one orbital period τ (TDAS) or multiples of it (ETDAS) in the past. In the calculation of the feedback signal, the CPU should use the suitable values of the control parameters (feedback gains k_i and memory parameters R_i) to achieve the stabilization of the tether libration motion towards the target periodic attitude motion. Finally, the CPU would periodically send the appropriate commands to the tether actuator devices to produce the adequate control torques. These actuators could be attitude thrusters or momentum wheels located at the end subsatellites. It is worth recalling here that, the control forces required in both methods will decrease to zero as the tether libration motion approaches to the target periodic motion.

5. Conclusions

In this work, we have studied the application of two feedback control methods, TDAS and ETDAS, to control the libration motion of an electrodynamic tether in an inclined elliptic orbit. The tether system considered in this study is based on the classical dumbbell model. The tether current is assumed constant and uniform along the tether length. The Earth′s magnetic field is modeled as a magnetic dipole aligned with the rotation axis of the Earth. It is also assumed that the tether trajectory is a frozen elliptic orbit not affected by the attitude motion.

By means of numerical continuation of families of periodic orbits, we have computed, in the uncontrolled case, the basic periodic libration motions with the same period as the orbital motion. All of these basic periodic motions are unstable for any values of the free parameters of the system: the orbital inclination i, the eccentricity e, and the electrodynamic parameter ε. The basic idea is to use these periodic attitude motions as the starting point for the operation of the tether, trying to stabilize those periodic motions by the application of two feedback control methods based on the time-delayed autosynchronization of the system.

Numerical simulations of the controlled libration motion of the tether have shown that the ability of both control techniques to get the stabilization of the basic periodic attitude motions depends on the values of the system parameters. In order to compare the relative effectiveness of both control methods, a stability analysis of the controlled periodic motions has been performed using an alternative technique suitable for these time-delayed feedback schemes. The control domains of both methods have been calculated for several values of the system parameters, as functions of the two control parameters (k_θ, k_φ) shared by both control methods. From the control domains we have computed the corresponding control success rates of both control techniques. The comparison of those success rates has shown that the ETDAS control scheme is slightly more effective than the

TDAS method in the stabilization of the periodic attitude motions. However, the more simplicity of the TDAS method makes it preferable to the ETDAS technique. This more complex method should be only applied in those cases of small eccentricity in which the TDAS technique fails.

The analysis carried out in this paper should be extended by the improvement of the tether model used in this study. In this way, some of the simplifications assumed here, as the rigid conductive tether, the constant tether current, the geomagnetic field model, or the frozen orbit, may be modified in order to study a more realistic model of the system. Work along this research line is now under consideration.

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