From Topological Dynamics to Discrete Markus-Yamabe Problem

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Global behavior of continuous and discrete dynamical systems is an interesting question. For local studies, Poincaré-Bendixon Theorem is a very powerful tool for continuous dynamical systems defined in $\mathbb{R}^2$, but in the case of discrete dynamical system is not so. It works if, for instance, we consider the time T-map of a flow given by a vector field. But what happens if the map is not even differentiable in $\mathbb{R}^2$? In this case chaotic behavior can appear.

First, we will see the relevance of Brower's Lemma of Translation Arcs and Degree Theory in describing the local dynamics of a fixed point and we will give some global dynamic configurations of orientation preserving homeomorphisms defined in $\mathbb{R}^2$.

Secondly, we will prove the existence of a global asymptotically attacting fixed point for continuous and injective maps of the plane (not necessarily homeomorphisms).

Finally we will see the relationship between these results and Markus-Yamabe Conjecture in dimension two.