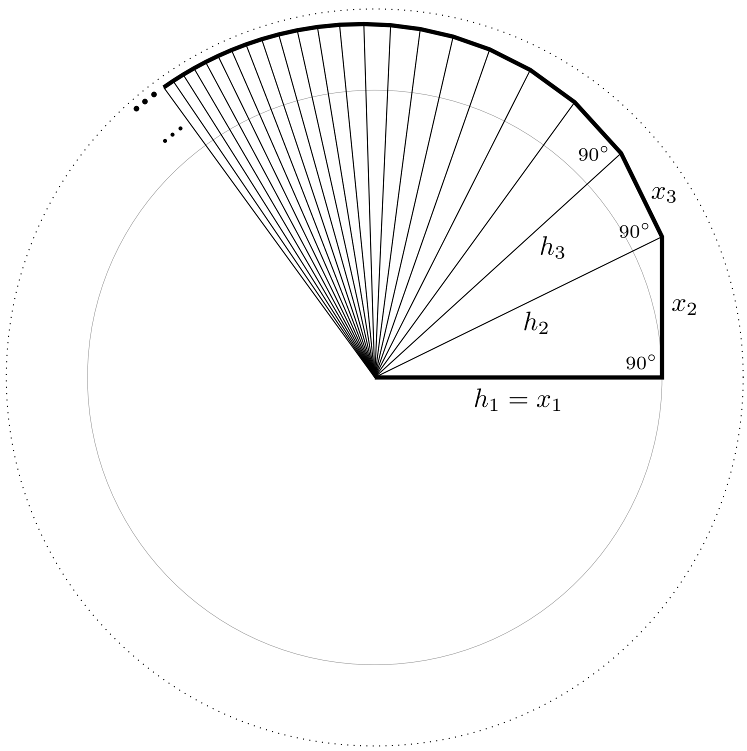
Proof Without Words: $\ell^1(\mathbb{R})$ is a subset of $\ell^2(\mathbb{R})$

JUAN LUIS VARONA
Universidad de La Rioja
26006 Logroño, Spain
jvarona@unirioja.es

Let's take the sequence $\mathbf{x} = \{x_n\}_{n=1}^{\infty}$, with $x_n > 0$ for every n. We define a new sequence $\{h_n\}_{n=1}^{\infty}$ as follows.

$$h_1 = x_1, \quad h_n^2 = h_{n-1}^2 + x_n^2, \quad n \geq 2 \quad \Rightarrow \quad h_n = \sqrt{x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2}$$



Because

$$\text{Length(spiral)} = \|\mathbf{x}\|_{\ell^1} \quad \text{and} \quad \text{Radius(asymptotic outer circle)} = \|\mathbf{x}\|_{\ell^2}$$

then

$$\text{Length(spiral)} < \infty \quad \Rightarrow \quad \text{Radius(asymptotic outer circle)} < \infty$$

or

$$\ell^1(\mathbb{R}) \subseteq \ell^2(\mathbb{R}).$$

Summary. We prove that $\ell^1(\mathbb{R})$ is a subset of $\ell^2(\mathbb{R})$ by means of a spiral.

JUAN LUIS VARONA (MR Author ID: 260232) is professor in the Department of Mathematics and Computation in the University of La Rioja, in Spain. He has published more that 50 research papers as well as an undergraduate book on number theory. He is editor in-chief of La Gaceta de la Real Sociedad Matemática Española.

Math. Mag. **90** (2017) 58–58. `doi:10.4169/math.mag.90.1.58`.
MSC: Primary 46B45, Secondary 40A05.