Mathematical Knowledge Management in Algebraic Topology

Jónathan Heras
Department of Mathematics and Computer Science
University of La Rioja
Logroño, Spain, 26004
jonathan.heras@unirioja.es

Author: Jónathan Heras
Institution: Department of Mathematics and Computer Science. University of La Rioja
Thesis Advisors: Vico Pascual and Julio Rubio
Committee Members: Eladio Domínguez (chair), Francis Sergeraert, Laurence Rideau, Francisco Jesús Martín-Mateos, Laureano Lambán
Title: Mathematical Knowledge Management in Algebraic Topology
Defended: May 2011

The work presented in this thesis tries to particularize Mathematical Knowledge Management to Algebraic Topology.

Mathematical Knowledge Management is a branch of Computer Science whose main goal consists in developing integral assistants for Mathematics including computation, deduction and powerful user interfaces able to make the daily work of mathematical researchers easier. Our application context is Algebraic Topology using the Kenzo system [1], a Common Lisp program devoted to Algebraic Topology developed by Francis Sergeraert, as an instrumental tool.

We can split the work presented in this thesis into three main parts which coincide with the main goals of Mathematical Knowledge Management.

Our first task has consisted in developing a system called fKenzo [2], an acronym of friendly Kenzo. This system not only provides a friendly graphical user interface to interact with the Kenzo system (the kernel of our application) but also guides the interaction of the user with the system (avoiding in this way execution errors). Moreover, fKenzo allows one to integrate other symbolic computation systems (such as GAP) and theorem prover tools (for instance, ACL2) by means of a plug-in system.

The second part of the thesis is focussed on increasing the computational capabilities of the Kenzo system. Three new Kenzo modules have been developed which in turn extend the fKenzo system. The first one allows us to study the pushout of simplicial sets, an important construction
which is involved in several usual Algebraic Topology constructions. The second one implements
the simplicial complex notion (a generalization of the graph notion to higher dimensions). The last
module allows us to analyse properties of 2D and 3D images by means of the Kenzo system thanks
to the computation of the homology groups associated with the image.

Finally, since the Kenzo system has obtained some results not confirmed nor refuted by any other
means, we are interested in increasing the reliability of the Kenzo system by means of Theorem
Proving tools. Namely, in our work we have used the ACL2 Theorem Prover [3]. ACL2 allows us
to prove properties of programs implemented in Common Lisp, as in the Kenzo case. Then, in our
work we have focussed on the certification of the correctness of some important fragments of the
Kenzo system and also the new modules developed in the second part of the thesis.

References

