Mathematical Knowledge Management in Algebraic Topology

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Title: Mathematical Knowledge Management in Algebraic Topology

Defended: May 2011

The work presented in this thesis tries to particularize Mathematical Knowledge Management to Algebraic Topology.

Mathematical Knowledge Management is a branch of Computer Science whose main goal consists in developing integral assistants for Mathematics including computation, deduction and powerful user interfaces able to make the daily work of mathematical researchers easier. Our application context is Algebraic Topology using the *Kenzo* system [1], a Common Lisp program devoted to Algebraic Topology developed by Francis Sergeraert, as an instrumental tool.

We can split the work presented in this thesis into three main parts which coincide with the main goals of Mathematical Knowledge Management.

Our first task has consisted in developing a system called *fKenzo* [2], an acronym of friendly *Kenzo*. This system not only provides a friendly graphical user interface to interact with the *Kenzo* system (the kernel of our application) but also guides the interaction of the user with the system (avoiding in this way execution errors). Moreover, *fKenzo* allows one to integrate other symbolic computation systems (such as *GAP*) and theorem prover tools (for instance, *ACL2*) by means of a plug-in system.

The second part of the thesis is focussed on increasing the computational capabilities of the *Kenzo* system. Three new *Kenzo* modules have been developed which in turn extend the *fKenzo* system. The first one allows us to study the pushout of simplicial sets, an important construction

which is involved in several usual Algebraic Topology constructions. The second one implements the simplicial complex notion (a generalization of the graph notion to higher dimensions). The last module allows us to analyse properties of 2D and 3D images by means of the *Kenzo* system thanks to the computation of the homology groups associated with the image.

Finally, since the *Kenzo* system has obtained some results not confirmed nor refuted by any other means, we are interested in increasing the reliability of the *Kenzo* system by means of Theorem Proving tools. Namely, in our work we have used the *ACL2* Theorem Prover [3]. *ACL2* allows us to prove properties of programs implemented in Common Lisp, as in the *Kenzo* case. Then, in our work we have focussed on the certification of the correctness of some important fragments of the *Kenzo* system and also the new modules developed in the second part of the thesis.

References

[1] X. Dousson, J. Rubio, F. Sergeraert, and Y. Siret. The *Kenzo* program. Institut Fourier, Grenoble, 1998. `http://www-fourier.ujf-grenoble.fr/~sergerar/Kenzo/`.

[2] J. Heras, V. Pascual, J. Rubio, and F. Sergeraert. *fKenzo*: A user interface for computations in Algebraic Topology. *Journal of Symbolic Computation*, 46:685–698, 2011.

[3] M. Kaufmann and J S. Moore. *ACL2*. `http://www.cs.utexas.edu/users/moore/acl2/`.