fKenzo: a user interface for computations in Algebraic Topology

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Preliminaries

- Kenzo:
  - Symbolic Computation System devoted to Algebraic Topology
  - Homology groups unreachable by any other means
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  - Not necessary to know Common Lisp

Kenzo and fKenzo as learning tools

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  - All the spaces (except S^2) are not of finite type
  - Challenging task, but easy with our systems

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  user interface

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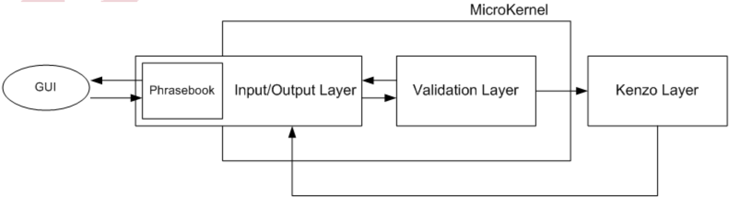

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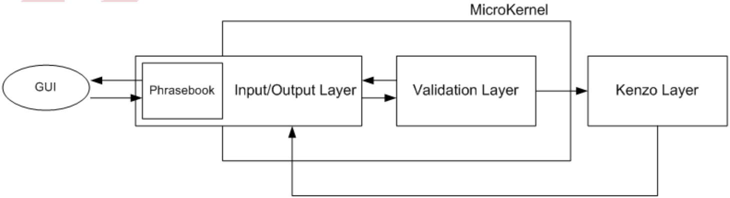

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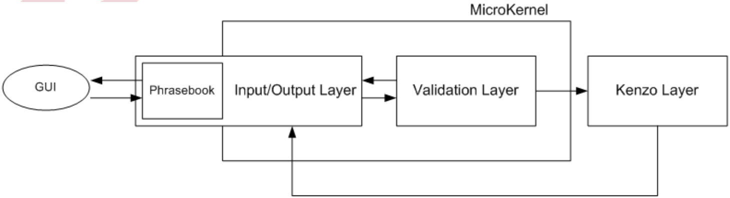

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fKenzo as CAS and TP framework

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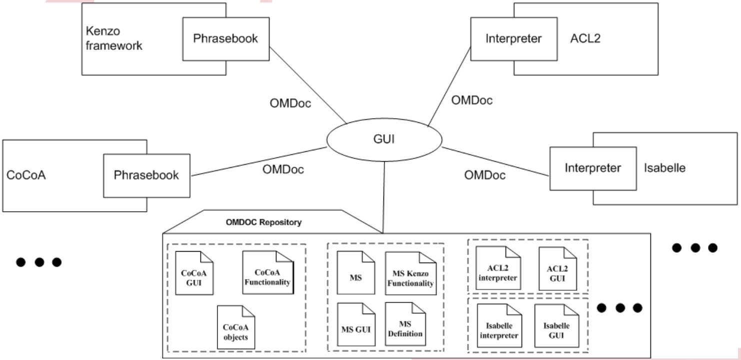

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  - three levels of information:
    - formulæ
    - mathematical statements
    - mathematical theories

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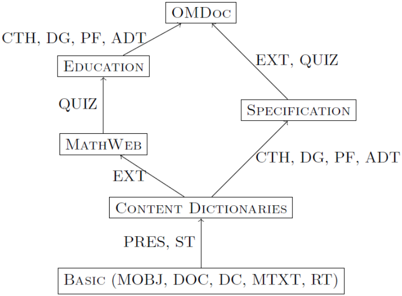

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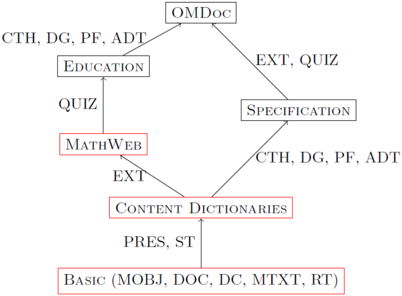

OMDoc Documents

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  - three levels of information:
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    - mathematical theories
- Sub-languages:
- 3 kinds of OMDoc documents:
  - Definition of Mathematical Structures
  - Functionality of the System
  - Definition of GUI structure

Definition of Mathematical Structures

- Goal:
  - Define the mathematical structures of the Computer Algebra Systems

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- OpenMath CDs $\subset$ OMDoc CDs

Functionality of the System

- Defines the functionality of each module
- Several Aims:
  - Interaction with Computer Algebra Systems and Theorem Provers
  - Event handlers
  - Interpreter from OMDoc to Theorem Provers
  - ...

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- Sub-language:
    - MathWeb sub-language (EXT module)
        - <code> tag
        - Common Lisp code
        - code for different applications

Definition of GUI structure

- Goal:
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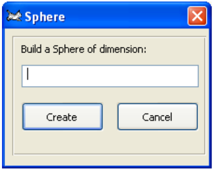
- XUL:
  - Mozilla's XML-based user interface language
  - To build feature rich cross-platforms




```
<OMForeign>
  <window name="Sphere">
    <groupbox>
      <label value="Build a Sphere of dimension:"/>
      <textbox id="n" type="number" min="1" max="14"/>
      <hbox>
        <button label="Create" onclick="create-sphere-on-click"/>
        <button label="Cancel" onclick="cancel-sphere-on-click"/>
      </hbox>
    </groupbox>
  </window>
</OMForeign>
```

Gathering all the pieces

- Goal:
  - Glue all the parts of a module

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```
<omdoc id="simplicial-sets.omdoc">
    ...
    <omgroup type="sequence">
        <ref xref="simplicial-sets-logic"/>
        <ref xref="simplicial-sets-presentation"/>
        <ref xref="simplicial-sets-conceptual-model"/>
    </omgroup>
    ...
</omdoc>
```

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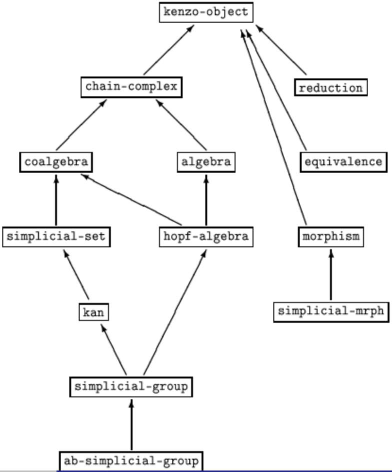
Kenzo Content Dictionaries

- Kenzo works with the main mathematical structures used in Simplicial Algebraic Topology



Organization of CDs

- All the mathematical structures Kenzo works with are graded structures.
- Each graded structure is represented in Kenzo by means of the invariant of its underlying set.

```
inv: U nat  -> bool
     x  n   -> True  if x ∈ K^n
               False if x ∉ K^n
```

Specification of a Mathematical Structure $\langle \Sigma, Prop \rangle$ $\rightarrow$ Specification of a Mathematical Structure *Representation* $\langle \Sigma \cup \{inv\}, Prop \cup \{Prop_{inv}\} \rangle$

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Specification of a Mathematical Structure $\langle \Sigma, Prop \rangle$ $\rightarrow$ Specification of a Mathematical Structure *Representation* $\langle \Sigma \cup \{inv\}, Prop \cup \{Prop_{inv}\} \rangle$

- Each OpenMath Representation of a Mathematical Structure has:
  - Signature (in a Signature Dictionary)
  - Properties of the mathematical structure
  - Example
  - Predefined Objects (optional)

ACL2

- ACL2 (A Computational Logic for an Applicative Common Lisp)

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- Encapsulate: to the constrained introduction of new functions
  - Signatures
  - Properties
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From a Kenzo CD to an ACL2 encapsulate

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  - Witness
- Interpreter from Kenzo CDs to ACL2 Encapsulates

OMDoc CDs		ACL2 Encapsulates
Signatures	$\longrightarrow$	*Signatures*
Properties	$\longrightarrow$	*Properties*
Example	$\longrightarrow$	*Witness*

A case study: simplicial sets

Definition

A simplicial set K, is a disjoint union $K = \bigcup_{q \geq 0} K^q$, where the K^q are sets, together with functions

$$
\begin{array}{lll}
\partial_i^q : K^q \rightarrow K^{q-1}, & q > 0, & i = 0, \ldots, q, \\
\eta_i^q : K^q \rightarrow K^{q+1}, & q \geq 0, & i = 0, \ldots, q,
\end{array}
$$

subject to relations

$$
\begin{array}{lllr}
\partial_i^{q-1}\partial_j^q & = & \partial_{j-1}^{q-1}\partial_i^q, & i < j \\
\eta_i^{q+1}\eta_j^q & = & \eta_j^{q+1}\eta_{i-1}^q, & i > j \\
\partial_i^{q+1}\eta_j^q & = & \eta_{j-1}^{q-1}\partial_i^q, & i < j \\
\partial_i^{q+1}\eta_i^q & = & \partial_{i+1}^{q+1}\eta_i^q, & \text{identity} \\
\partial_i^{q+1}\eta_j^q & = & \eta_j^{q-1}\partial_{i-1}^q, & i > j+1
\end{array}
$$

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```
<Signature name="simplicial-set">
    <OMOBJ xmlns="http://www.openmath.org/OpenMath">
        <OMA>
            <OMS name="mapsto" cd="sts"/>
            <OMA id="inv">
                <OMS cd="sts" name="mapsto"/>
                <OMV name="Simplicial-Set-Element"/>
                <OMV name="PositiveInteger"/>
                <OMS cd="setname2" name="boolean"/>
            </OMA>
            <OMA id="face">
                <OMS cd="sts" name="mapsto"/>
                <OMV name="Simplicial-Set-Element"/>
                <OMV name="PositiveInteger"/>
                <OMV name="PositiveInteger"/>
                <OMV name="Simplicial-Set-Element"/>
            </OMA>
            <OMA id="degeneracy">
                <OMS cd="sts" name="mapsto"/>
                <OMV name="Simplicial-Set-Element"/>
                <OMV name="PositiveInteger"/>
                <OMV name="PositiveInteger"/>
                <OMV name="Simplicial-Set-Element"/>
            </OMA>
            <OMV name="Simplicial-Set"/>
        </OMA>
    </OMOBJ>
</Signature>
```

A case study: simplicial sets

```
(((face * * *) => *)
 ((degeneracy * * *) => *)
 ((inv * *) => *))
```

A case study: simplicial sets

```
<CMP> The face operator is well defined </CMP>
<FMP>
...
<OMA>
    <OMS cd="logic1" name="implies"/>
    <OMA>
        <OMV name="inv"/>
        <OMV name="x"/>
        <OMV name="q"/>
    </OMA>
    <OMA>
        <OMV name="inv"/>
        <OMA>
            <OMV name="face"/>
            <OMV name="x"/>
            <OMV name="i"/>
            <OMV name="q"/>
        </OMA>
        <OMA>
            <OMS cd="arith1" name="minus"/>
            <OMV name="q"/>
            <OMI>1</OMI>
        </OMA>
    </OMA>
</OMA>
...
</FMP>
```

A case study: simplicial sets

```
; The face operator is well defined
(defthm prop1
(implies (inv x q) (inv (face x i q) (- q 1))))
```

A case study: simplicial sets

```
<example>
...
    <OMBIND>
        <OMS name="face"/>
        <OMBVAR>
            <OMV name="x"/>
            <OMV name="i"/>
            <OMV name="q"/>
        </OMBVAR>
        <OMS cd="list" name="nil"/>
    </OMBIND>
...
</example>
```

A case study: simplicial sets

```
(local (defun face (x i q)
 (declare (IGNORE x i q))
 nil))
```

A case study: simplicial sets

Demo

GAP, Kenzo and ACL2

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- ACL2:
  - Theorem Prover
  - First order logic
  - Based on Common Lisp

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  Obtain resolutions of Cyclic Groups from HAP

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  6. Build the cyclic group in Kenzo
  7. Assign the resolution to the correspondent cyclic group in Kenzo

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  7. Assign the resolution to the correspondent cyclic group in Kenzo
  8. Build the space $K(G, 1)$ where G is the cyclic group in Kenzo

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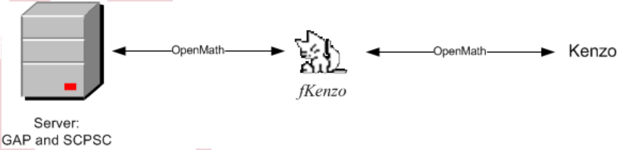
Connecting GAP, Kenzo and ACL2 II



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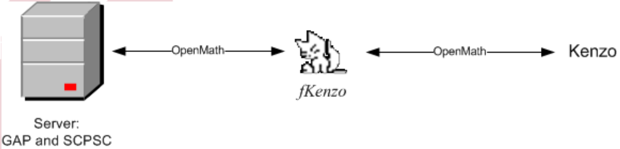

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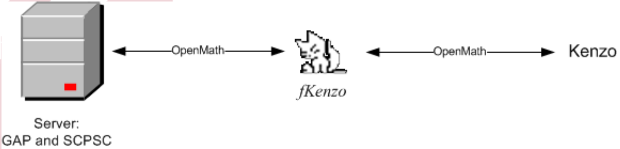

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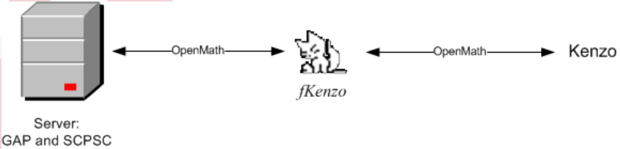

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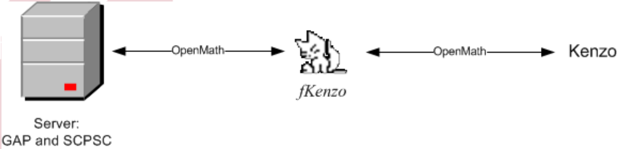

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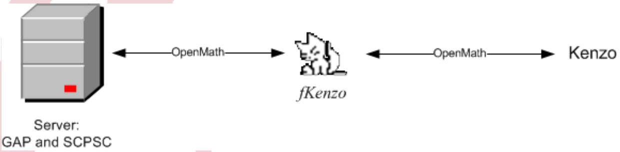

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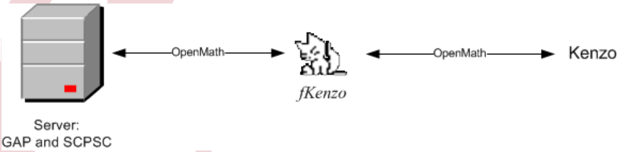

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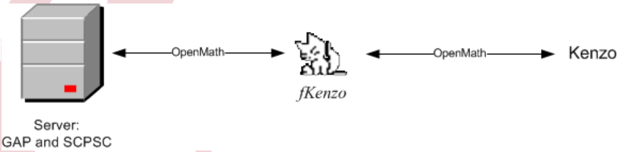

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`Mathematica.wmv`

Further Work

- Find a suitable way of editing spaces
- Extend fKenzo:
    - Kenzo evolves
    - Connection with other CAS and TP.

fKenzo: a user interface for computations in Algebraic Topology

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