Using Open Mathematical Documents to interface Computer Algebra and Proof Assistant systems

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Introduction

Kenzo:
- Symbolic Computation System devoted to Algebraic Topology
- Homology groups unreachable by any other means
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- Increase the reliability of Kenzo
  - Integration of Kenzo with ACL2
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  - Homology groups unreachable by any other means
- Goal:
  - Increase the reliability of Kenzo
    - Integration of Kenzo with ACL2
- Necessary:
  - Computation
  - Representation of the Mathematical Knowledge
  - Deduction
Kenzo + mediated access

Figure: Simplified diagram of the architecture
Introduction

Representation

- OpenMath: XML standard
Introduction

Representation

- OpenMath: XML standard
- Kenzo works with the main mathematical structures used in Simplicial Algebraic Topology
OpenMath: XML standard

Kenzo works with the main mathematical structures used in Simplicial Algebraic Topology

A CD without axioms for each Mathematical Structure was developed
Introduction

Deduction

- ACL2 (A Computational Logic for an Applicative Common Lisp)
Introduction

Deduction

- ACL2 (A Computational Logic for an Applicative Common Lisp)
- ACL2 Programming Language
- First-Order Logic
- Theorem Prover
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- Proof techniques:
  - Simplification
  - Induction
  - “The Method”
Deduction

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- ACL2
  - Programming Language
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- Proof techniques:
  - Simplification
  - Induction
  - "The Method"
- Encapsulate: to the constrained introduction of new functions
  - Signatures
  - Properties
  - Witness
A Graphical User Interface to gather all the pieces was developed

- Customizable by means of an OMDoc Document Repository
- OpenMath $\rightarrow$ OMDoc due to OMDoc tools
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OMDoc Documents

OMDoc format:
- mathematical documents + knowledge encapsulate in them
- three levels of information:
  - formulæ
  - mathematical statements
  - mathematical theories
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Sub-languages:
OMDoc Documents

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    - formulæ
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- Sub-languages:

```plaintext
Fig. 1 OMDoc Sub-Languages used
```

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Interfacing Computer Algebra and Proof Assistant systems
OMDoc Documents

OMDoc format:
- mathematical documents + knowledge encapsulate in them
- three levels of information:
  - formulæ
  - mathematical statements
  - mathematical theories

Sub-languages:

5 kinds of OMDoc documents:
- Definition of Mathematical Structures
- Logic to interact with Kenzo
- Presentation for the GUI
- ACL2 interpreter
- ACL2 presentation for the GUI
Definition of Mathematical Structures

Goal:
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  - Signature
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  - OMDoc Content Dictionaries for each mathematical structure:
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- OpenMath CDs ⊂ OMDoc CDs
Logic to interact with Kenzo

- Kenzo keeps on growing
- Goal:
  - Include new functionality
  - Extensibility of the system
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- Sub-language:
  - MathWeb sub-language (EXT module)
    - `<code>` tag
    - Common Lisp code
    - code for different applications
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    - Common Lisp code
    - code for different applications
- Functionality for each mathematical structure defined
Presentation for the GUI

Goal:
- Define the structure of our GUI
Presentation for the GUI

- **Goal:**
  - Define the structure of our GUI

- **Sub-language:**
  - MathWeb sub-language
    - `<OMForeign>` tag
    - XUL

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Presentation for the GUI

- **Goal:**
  - Define the structure of our GUI
- **Sub-language:**
  - MathWeb sub-language
  - `<OMForeign>` tag
  - XUL
- **XUL:**
  - Mozilla’s XML-based user interface language
  - To build feature rich cross-platforms

```
<OMForeign>
  <window name="Sphere">
    <groupbox>
      <label value="Build a Sphere of dimension:"/>
      <textbox id="n" type="number" min="1" max="14">
        1
      </textbox>
      <hbox>
        <button label="Create" onclick="create-sphere-on-click"/>
        <button label="Cancel" onclick="cancel-sphere-on-click"/>
      </hbox>
    </groupbox>
  </window>
</OMForeign>
```
ACL2 Interpreter

Goal:
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- By means of OMDoc CDs
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<table>
<thead>
<tr>
<th>OMDoc CDs</th>
<th>ACL2 Encapsulates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signatures</td>
<td>Signatures</td>
</tr>
<tr>
<td>Properties</td>
<td>Properties</td>
</tr>
<tr>
<td>Examples</td>
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</tr>
</tbody>
</table>
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![Image of Kenzo Interface GUI](image-url)
ACL2 presentation for the GUI

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Gathering all the pieces

Goal:
- Glue all the parts of a mathematical structure
Gathering all the pieces

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- Sub-language:
  - Basic OMDoc
  - `<omgroup>` tag

```xml
<omdoc id="simplicial-sets.omdoc">
  ...
  <omgroup type="sequence">
    <ref xref="simplicial-sets-logic"/>
    <ref xref="simplicial-sets-presentation"/>
    <ref xref="simplicial-sets-conceptual-model"/>
  </omgroup>
  ...
</omdoc>
```
Gathering all the pieces

- **Goal:**
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Workflow

Loading Simplicial Sets and ACL2 in our GUI:

Figure: Workflow diagram
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- Same OMDoc sub-language to reach different goals.
- Integration of representation, computation and deduction in the same system.

Further Work:
- Implement interesting interactions between ACL2 and Kenzo.
- Integration with other Symbolic Computation Systems.
Conclusions and Further Work

Further Work

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