Using Open Mathematical Documents to interface Computer Algebra and Proof Assistant systems

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Table of Contents

1. Introduction
2. Specifying with OMDoc Documents
3. Conclusions and Further Work

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1 Introduction

2 Specifying with OMDoc Documents

3 Conclusions and Further Work

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  - Homology groups unreachable by any other means
- Goal:
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    - Integration of Kenzo with ACL2
- Necessary:
  - Computation
  - Representation of the Mathematical Knowledge
  - Deduction

Computation

- Kenzo + mediated access

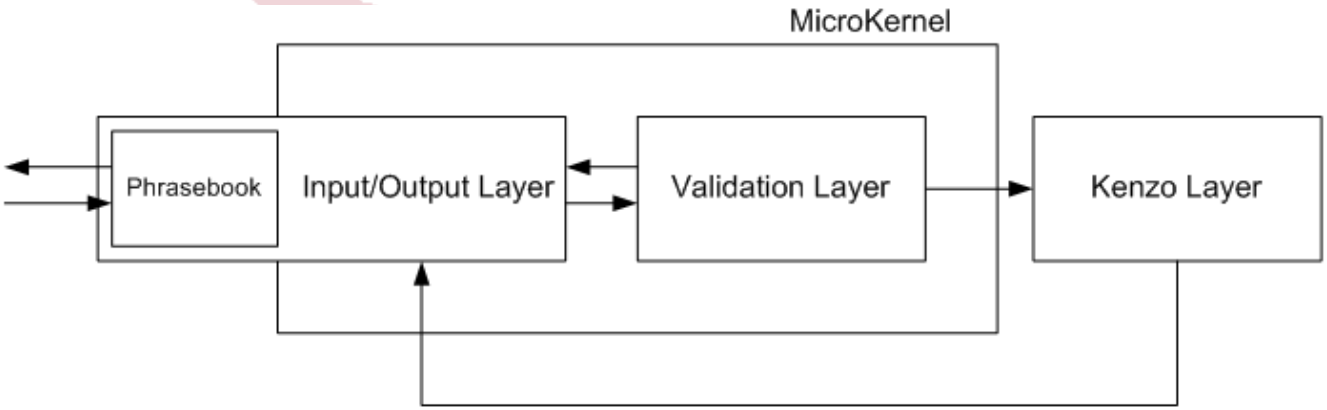

Figure: Simplified diagram of the architecture

Representation

- OpenMath: XML standard

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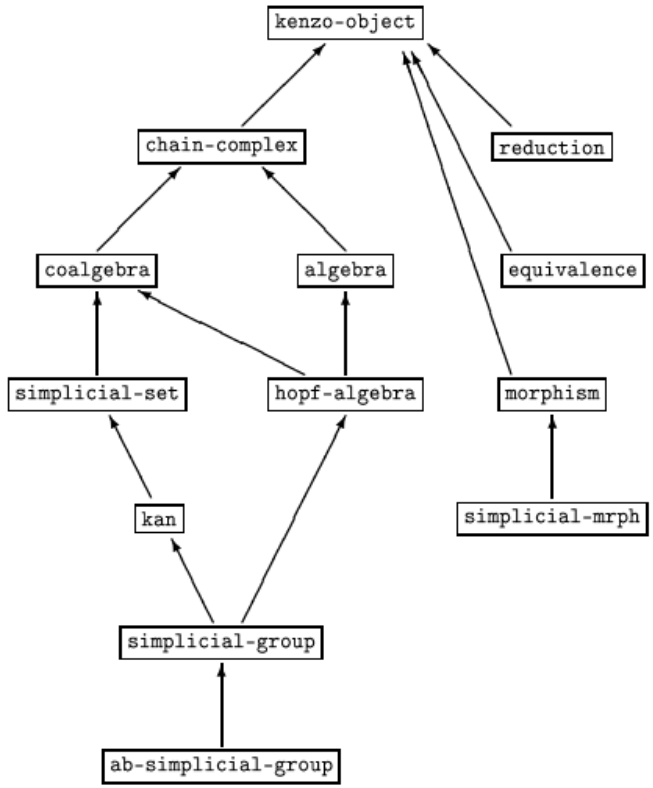

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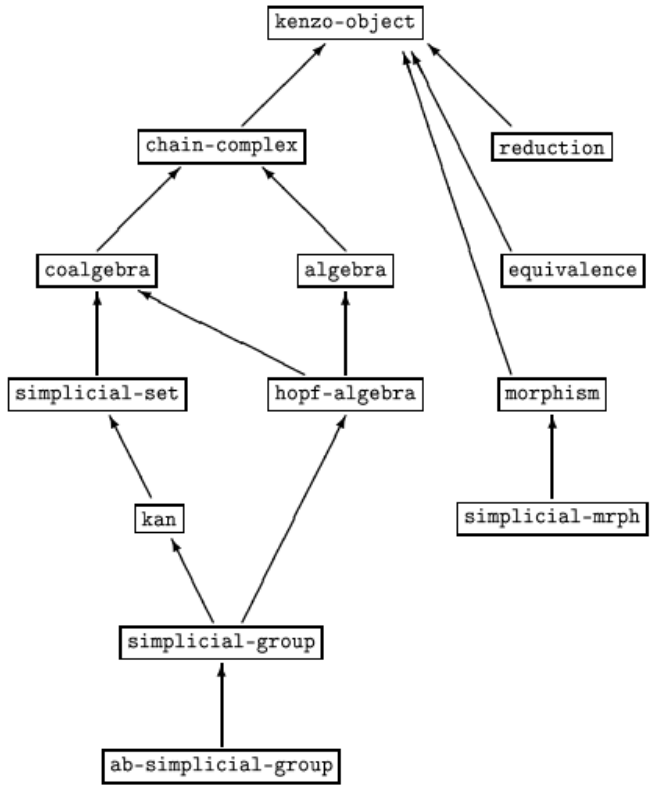

- A CD without axioms for each Mathematical Structure was developed

Deduction

- ACL2 (A Computational Logic for an Applicative Common Lisp)

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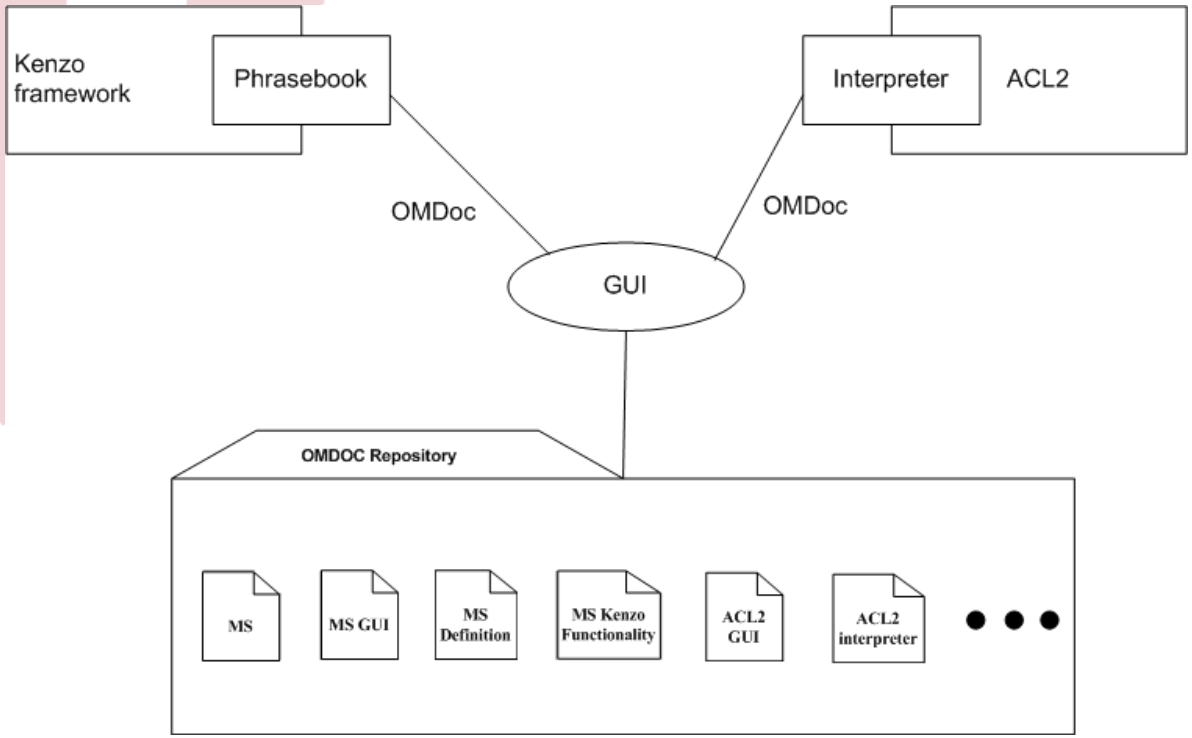
- Encapsulate: to the constrained introduction of new functions
  - Signatures
  - Properties
  - Witness

Gathering all the pieces



- A Graphical User Interface to gather all the pieces was developed
  - Customizable by means of an OMDoc Document Repository
  - OpenMath $\longrightarrow$ OMDoc due to OMDoc tools

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  - three levels of information:
    - formulæ
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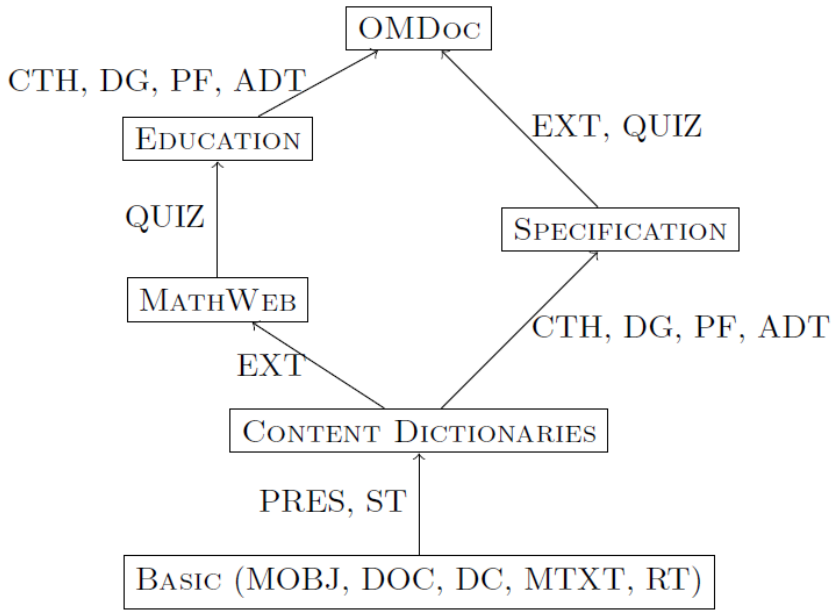

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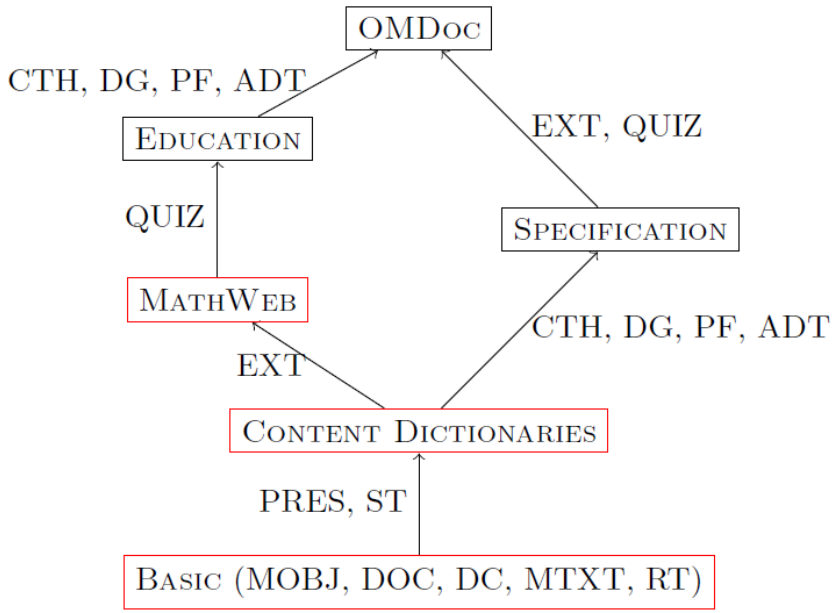

OMDoc Documents

- OMDoc format:
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  - three levels of information:
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- Sub-languages:
- 5 kinds of OMDoc documents:
  - Definition of Mathematical Structures
  - Logic to interact with Kenzo
  - Presentation for the GUI
  - ACL2 interpreter
  - ACL2 presentation for the GUI

Definition of Mathematical Structures

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- OpenMath CDs $\subset$ OMDoc CDs

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- Functionality for each mathematical structure defined

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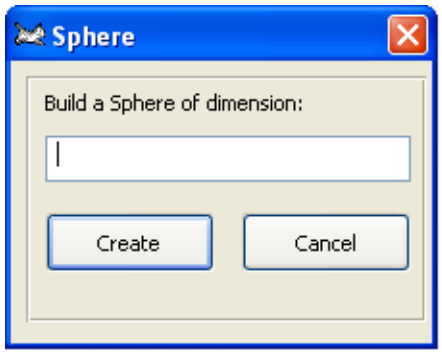
- XUL:
  - Mozilla's XML-based user interface language
  - To build feature rich cross-platforms




```
<OMForeign>
  <window name="Sphere">
    <groupbox>
      <label value="Build a Sphere of dimension:"/>
      <textbox id="n" type="number" min="1" max="14"/>
      <hbox>
        <button label="Create" onclick="create-sphere-on-click"/>
        <button label="Cancel" onclick="cancel-sphere-on-click"/>
      </hbox>
    </groupbox>
  </window>
</OMForeign>
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OMDoc CDs		ACL2 Encapsulates
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Properties	$\longrightarrow$	*Properties*
Examples	$\longrightarrow$	*Witness*

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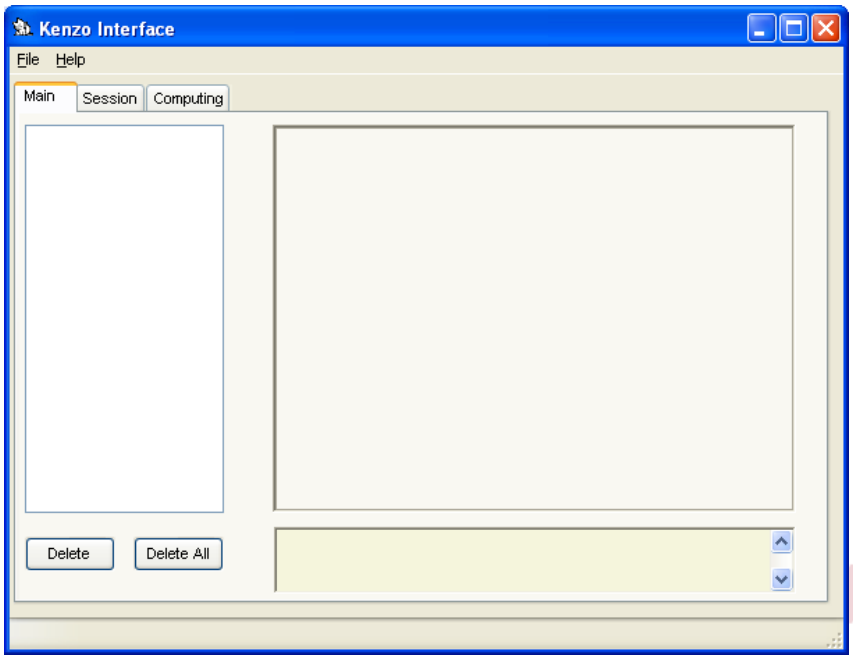

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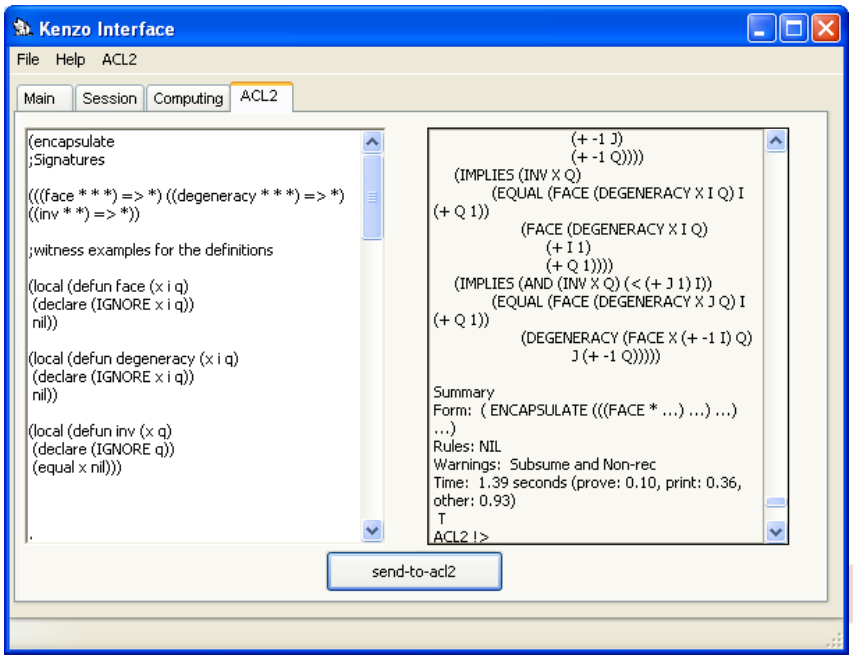

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```
<omdoc id="simplicial-sets.omdoc">
    ...
    <omgroup type="sequence">
        <ref xref="simplicial-sets-logic"/>
        <ref xref="simplicial-sets-presentation"/>
        <ref xref="simplicial-sets-conceptual-model"/>
    </omgroup>
    ...
</omdoc>
```

Workflow

Loading Simplicial Sets and ACL2 in our GUI:

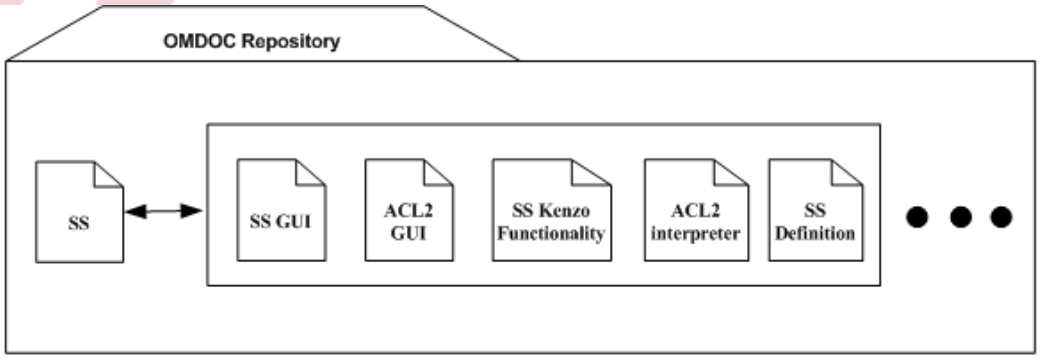

Figure: Workflow diagram

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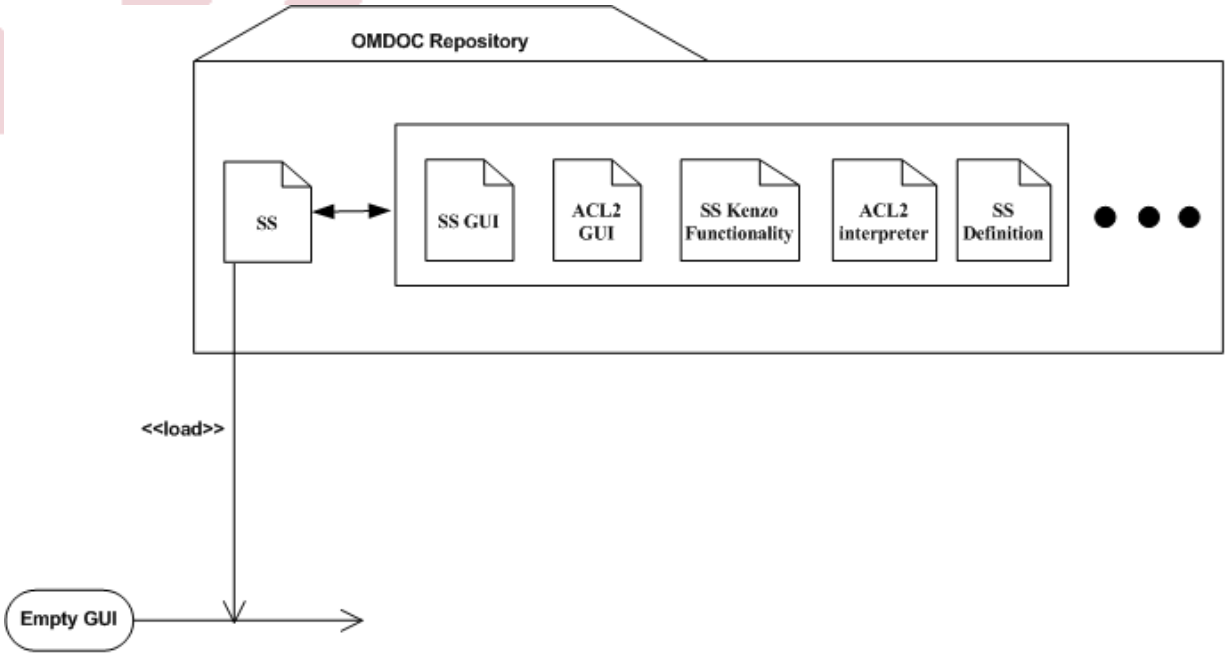

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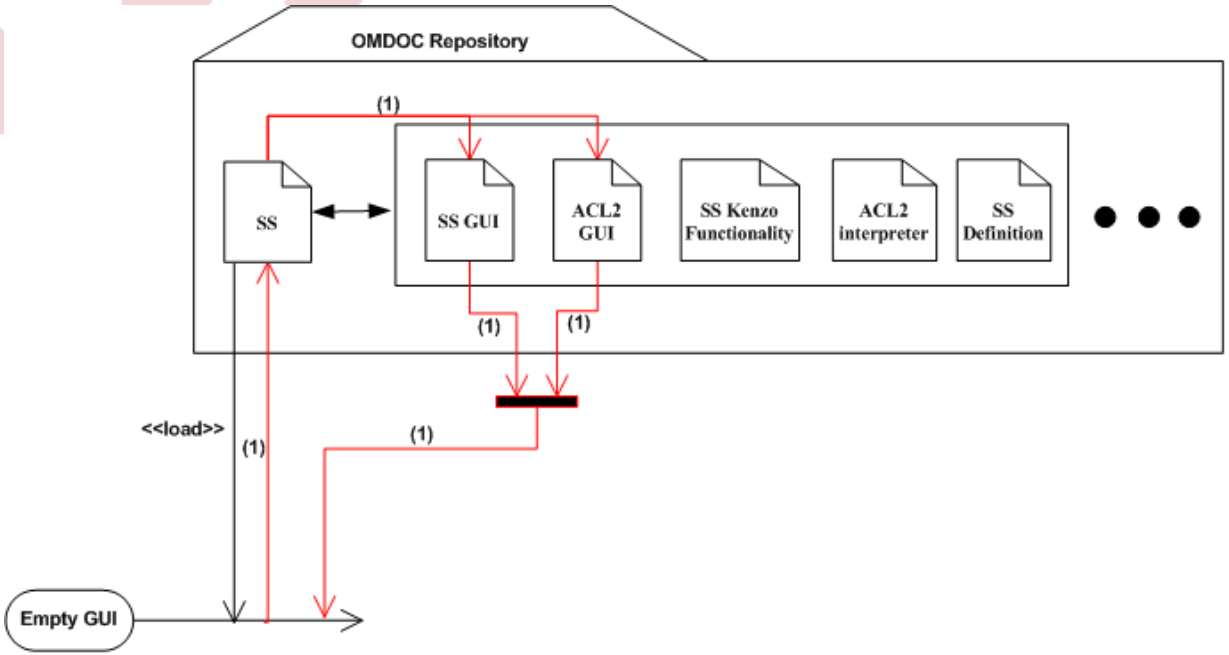

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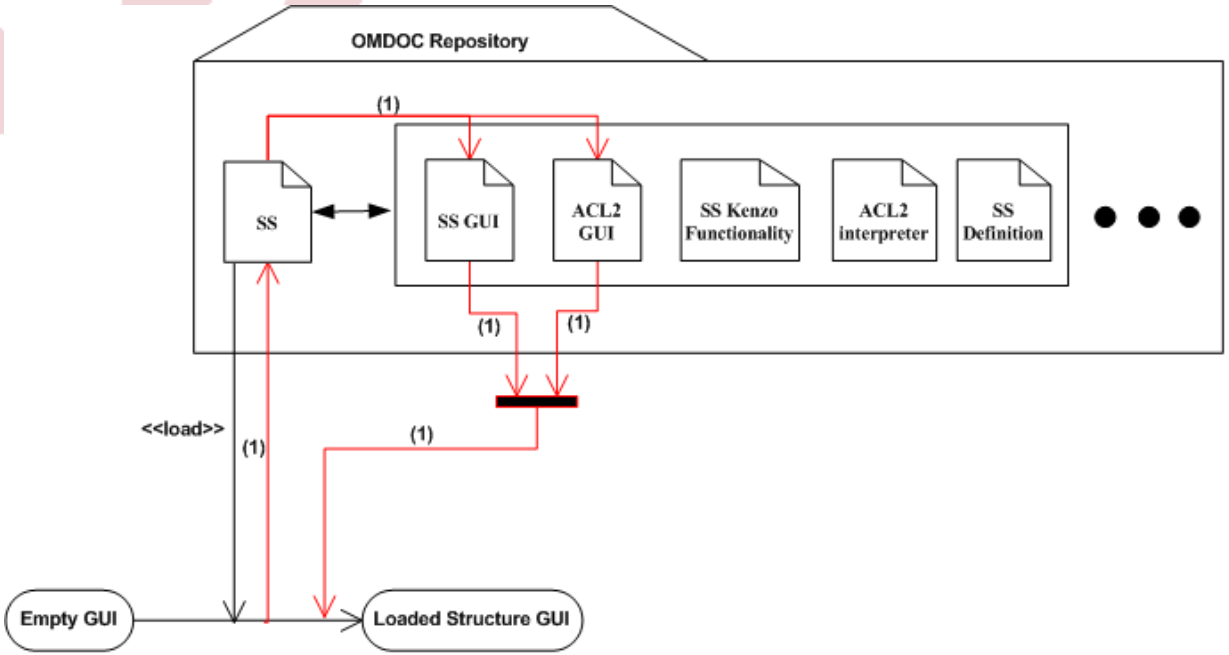

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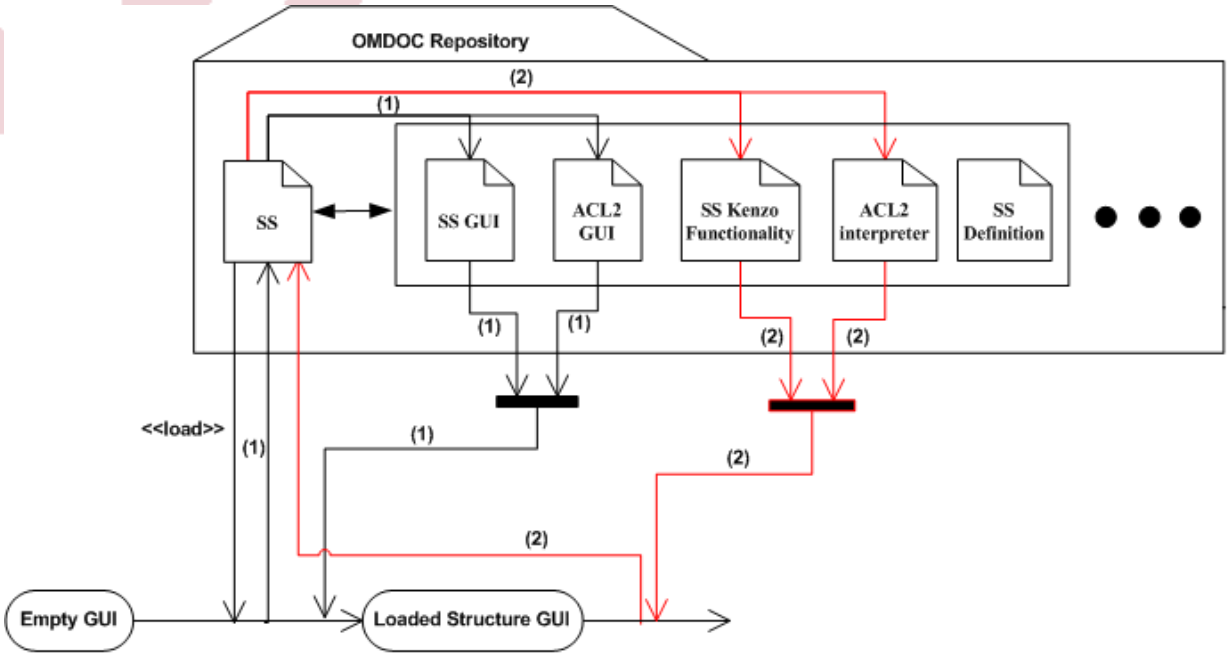

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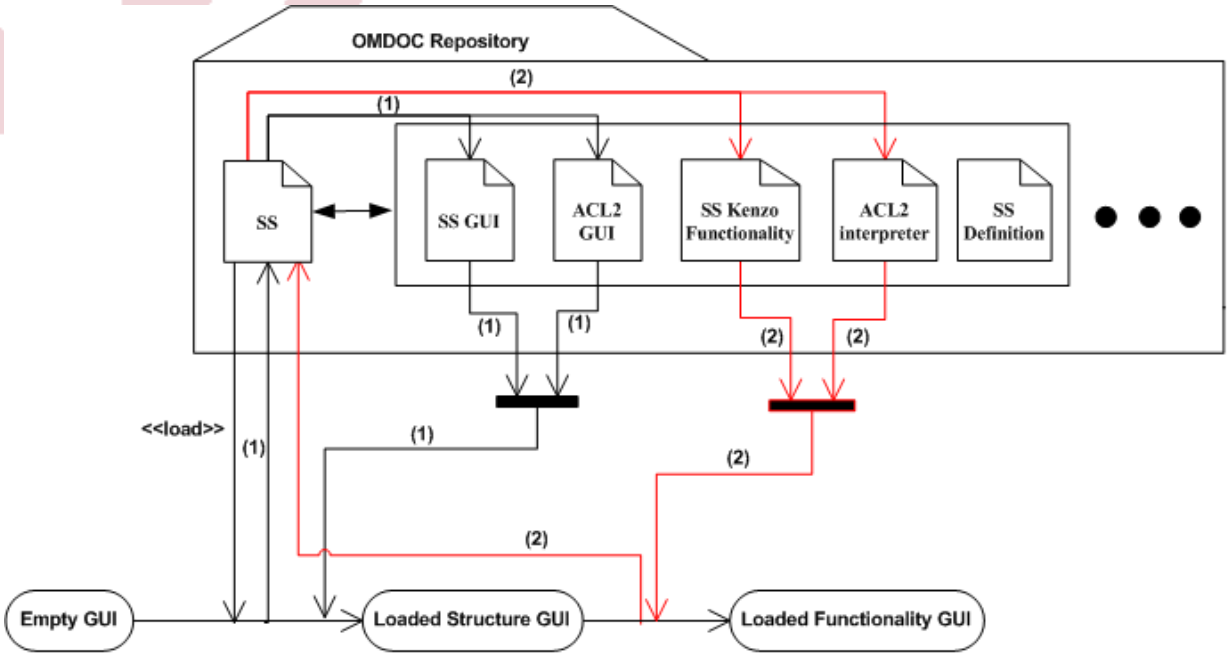

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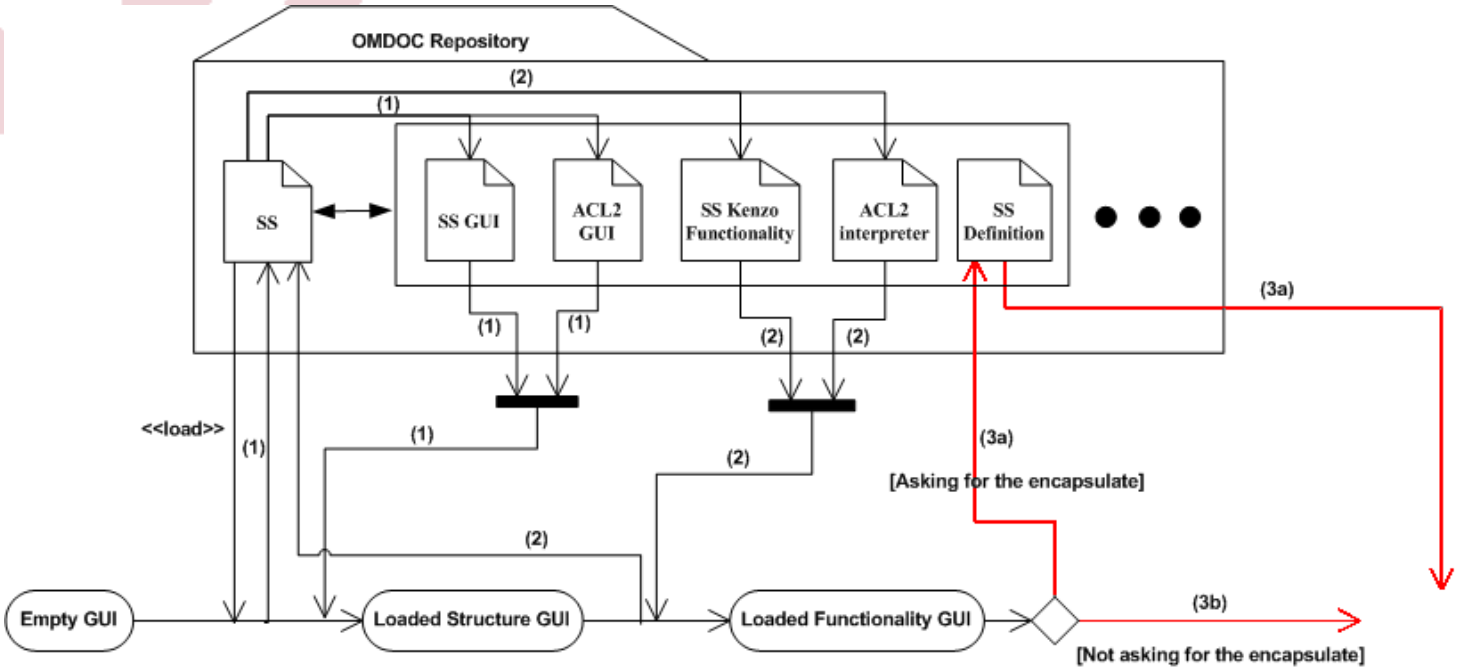

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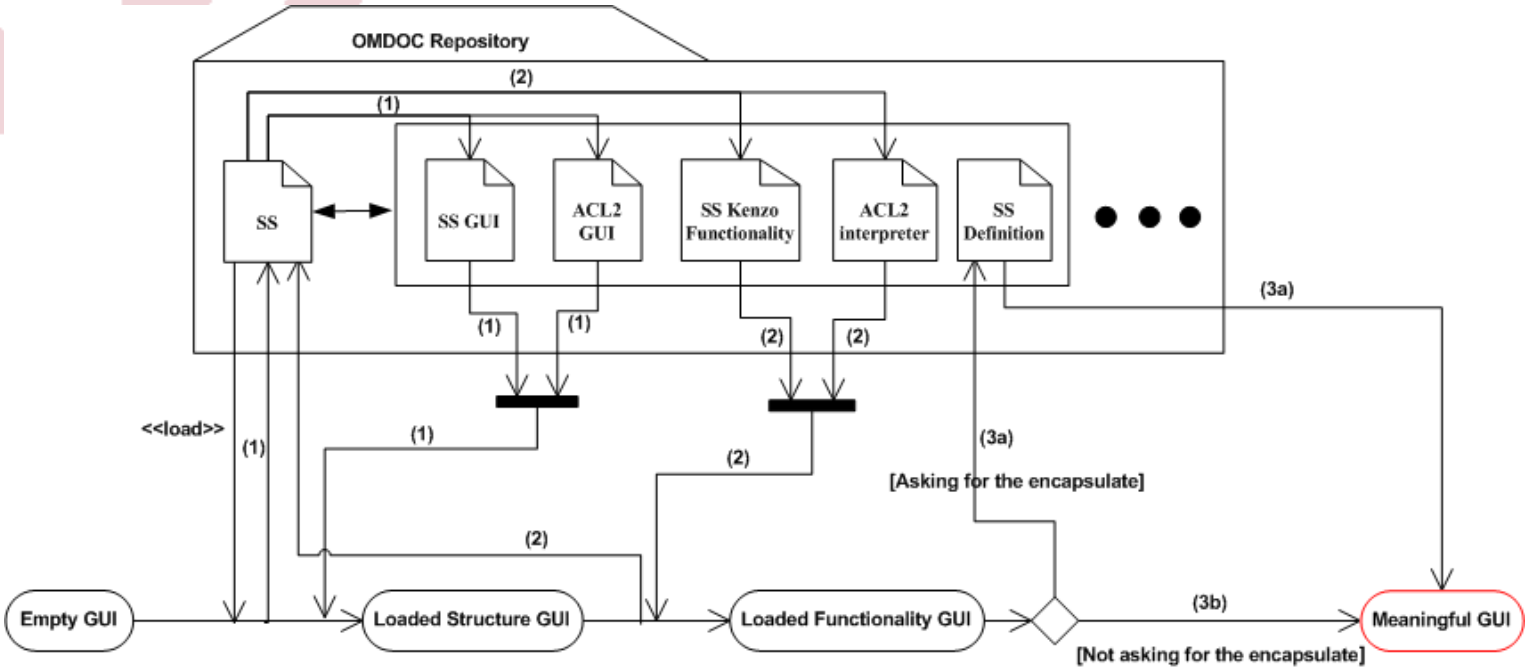

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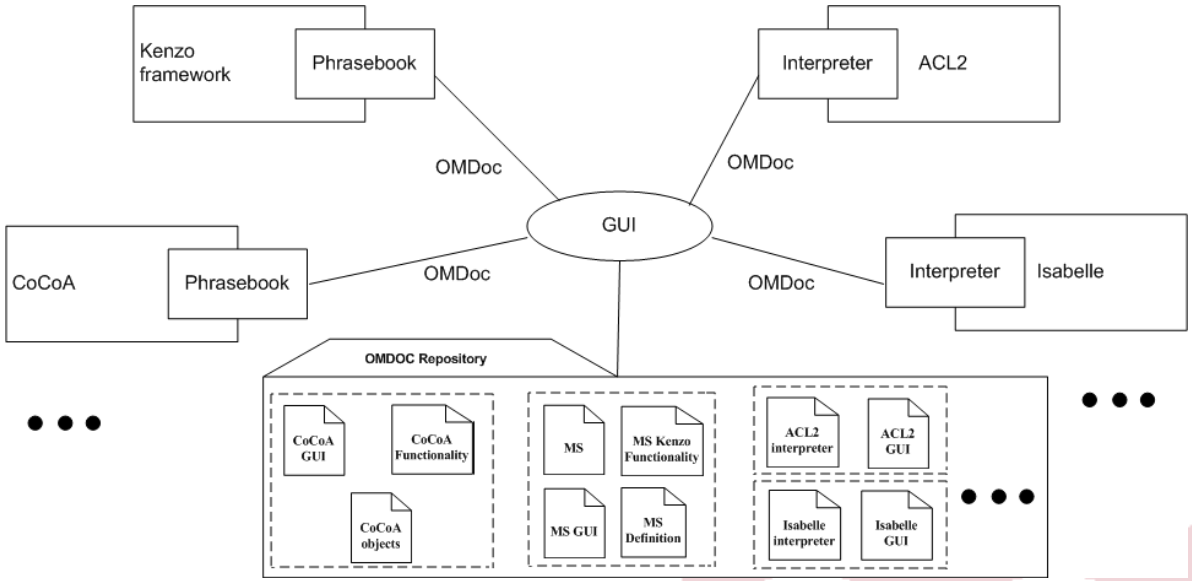

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