Neuronal structure detection using Persistent Homology*

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Outline

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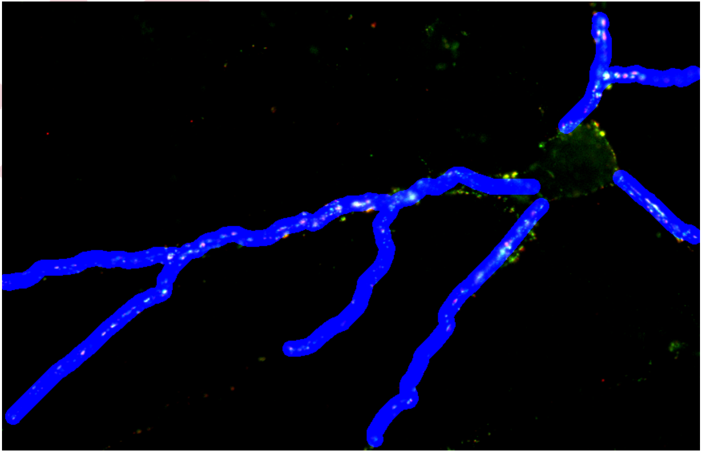
Motivation



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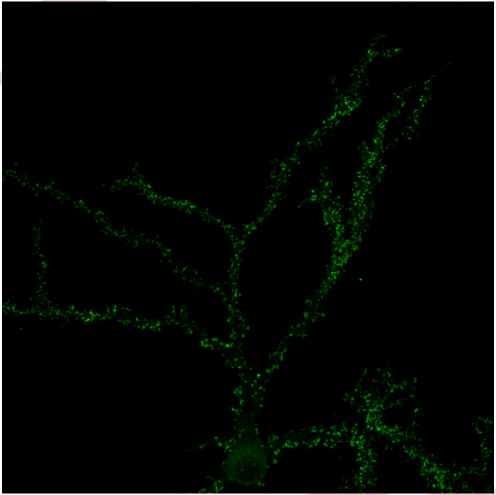

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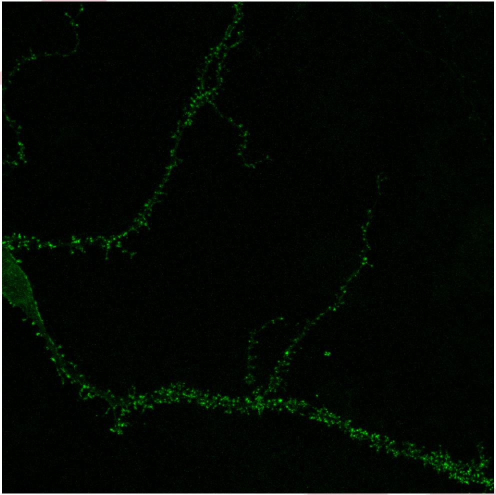

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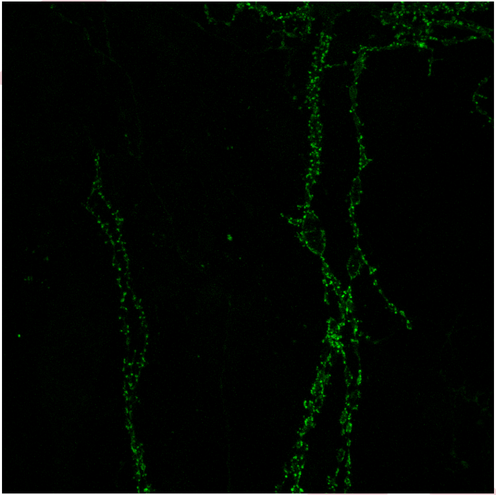
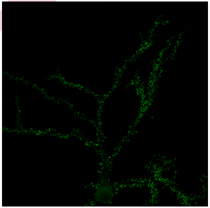
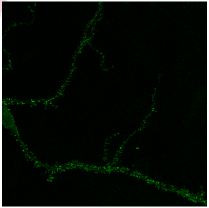
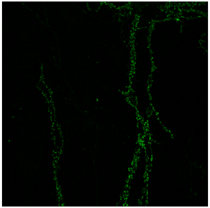


Motivation

Peculiarity: the last three images are obtained from a stack

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Intuitive idea

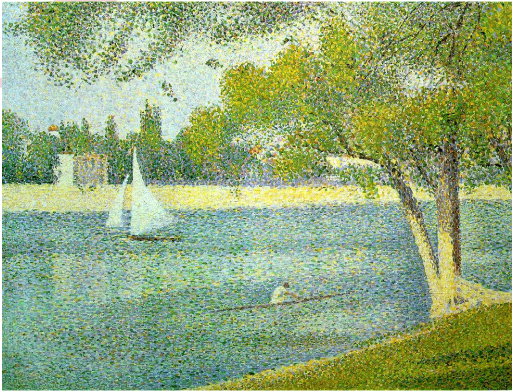

Figure: La Seine à la Grande-Jatte. Seurat, Georges

Key ideas

Persistence key ideas:

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- Provide an abstract framework to:
  - Measure scales on topological features
  - Order topological features in term of importance/noise
- How long is a topological feature persistent?
  - As long as it refuses to die . . .
- Roughly speaking:
  - Homology detects topological features (connected components, holes, and so on)
  - Persistent homology describes the evolution of topological features looking at consecutive thresholds

History

- Biogeometry project of Edelsbrunner
  - C. J. A. Delfinado and H. Edelsbrunner. *An incremental algorithm for Bettin numbers of simplicial complexes on the 3-sphere*. Computer Aided Geometry Design 12 (1995):771–784.
- Work of Frosini, Ferri and collaborators
  - P. Frosini and C. Landi. *Size theory as a topological tool for computer vision*. Pattern Recognition and Image Analysis 9 (1999):596–603.
- Doctoral thesis of Robins
  - V. Robins. *Toward computing homology from finite approximations*. Topology Proceedings 24 (1999):503–532.

Simplicial Complexes

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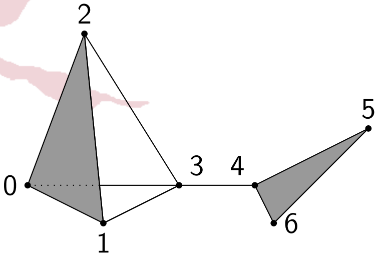
Definition

An ordered (abstract) simplicial complex over V is a set of simplices $\mathcal{K}$ over V satisfying the property:

$$\forall \alpha \in \mathcal{K}, \text{ if } \beta \subseteq \alpha \Rightarrow \beta \in \mathcal{K}$$

Let $\mathcal{K}$ be a simplicial complex. Then the set $S_n(\mathcal{K})$ of n-simplices of $\mathcal{K}$ is the set made of the simplices of cardinality $n + 1$.

An example



$V = (0, 1, 2, 3, 4, 5, 6)$
$\mathcal{K} = \{\emptyset, (0), (1), (2), (3), (4), (5), (6),$
$(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3), (3, 4), (4, 5), (4, 6), (5, 6),$
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Chain Complexes

Definition

A chain complex C_* *is a pair of sequences* $C_* = (C_q, d_q)_{q\in\mathbb{Z}}$ *where:*

- *For every* $q \in \mathbb{Z}$*, the component* C_q *is an R-module, the chain group of degree* q
- *For every* $q \in \mathbb{Z}$*, the component* d_q *is a module morphism* $d_q : C_q \to C_{q-1}$*, the differential map*
- *For every* $q \in \mathbb{Z}$*, the composition* $d_q d_{q+1}$ *is null:* $d_q d_{q+1} = 0$

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Definition

Let $\mathcal{K}$ be an (ordered abstract) simplicial complex. Let $n \geq 1$ and $0 \leq i \leq n$ be two integers n and i. Then the face operator ∂_i^n is the linear map $\partial_i^n : S_n(\mathcal{K}) \to S_{n-1}(\mathcal{K})$ defined by:

$$\partial_i^n((v_0, \dots, v_n)) = (v_0, \dots, v_{i-1}, v_{i+1}, \dots, v_n).$$

The i-th vertex of the simplex is removed, so that an $(n-1)$-simplex is obtained.

Definition

Let $\mathcal{K}$ be a simplicial complex. Then the chain complex $C_*(\mathcal{K})$ canonically associated with $\mathcal{K}$ is defined as follows. The chain group $C_n(\mathcal{K})$ is the free $\mathbb{Z}$ module generated by the n-simplices of $\mathcal{K}$. In addition, let $(v_0, \dots, v_{n-1})$ be a n-simplex of $\mathcal{K}$, the differential of this simplex is defined as:

$$d_n := \sum_{i=0}^{n} (-1)^i \partial_i^n$$

Homology

Definition

If $C_* = (C_q, d_q)_{q \in \mathbb{Z}}$ is a chain complex:

- *The image $B_q = im\ d_{q+1} \subseteq C_q$ is the (sub)module of q-boundaries*
- *The kernel $Z_q = ker\ d_q \subseteq C_q$ is the (sub)module of q-cycles*

Given a chain complex $C_* = (C_q, d_q)_{q \in \mathbb{Z}}$:

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Let $C_* = (C_q, d_q)_{q \in \mathbb{Z}}$ be a chain complex. For each degree $n \in \mathbb{Z}$, the n-homology module of C_* is defined as the quotient module

$$H_n(C_*) = \frac{Z_n}{B_n}$$

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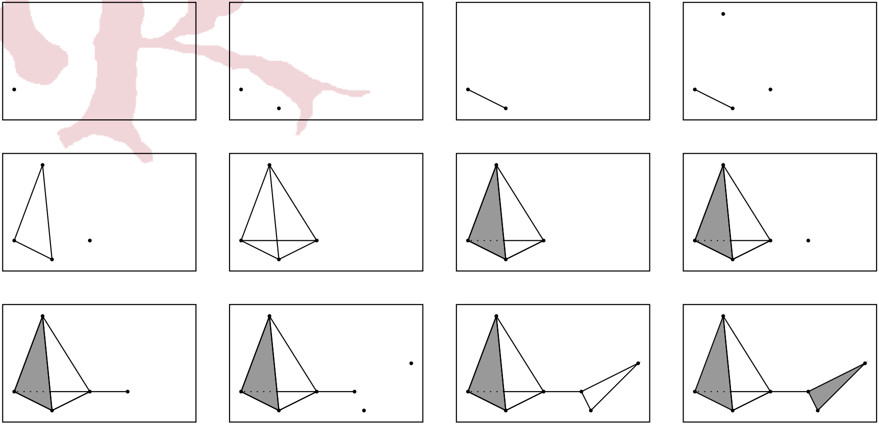
A filtration of a simplicial complex $\mathcal{K}$ is a nested subsequence of complexes

$$K^0 \subseteq K^1 \subseteq \ldots \subseteq K^m = \mathcal{K}$$

An example



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A filtered complex $\mathcal{K}$ with inclusion maps for the simplices becomes a persistence complex

$$
\begin{array}{ccccccc}
\downarrow{\scriptstyle d_3^0} & & \downarrow{\scriptstyle d_3^1} & & \downarrow{\scriptstyle d_3^2} & & \\
C_2^0 & \xrightarrow{f^0} & C_2^1 & \xrightarrow{f^1} & C_2^2 & \xrightarrow{f^2} & \cdots \\
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Let $\mathcal{C} = \{C_*^i\}_{i \geq 0}$ *be a persistence complex associated with a filtration. The p-persistent kth homology group of* $\mathcal{C}$ *is:*

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The p-persistent k-th Betti number $\beta_k^{i,p}$ is the rank of $H_k^{i,p}$

Persistent Homology groups

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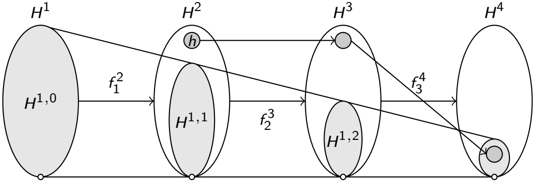

$h \in H^2$ is born in H^2, and dies entering H^4

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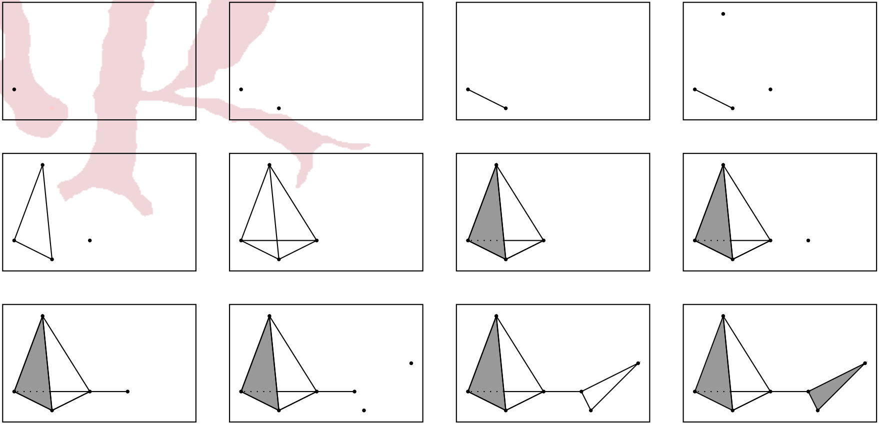
Finite multisets of P-intervals are plotted as disjoint unions of intervals, called barcodes.

Example



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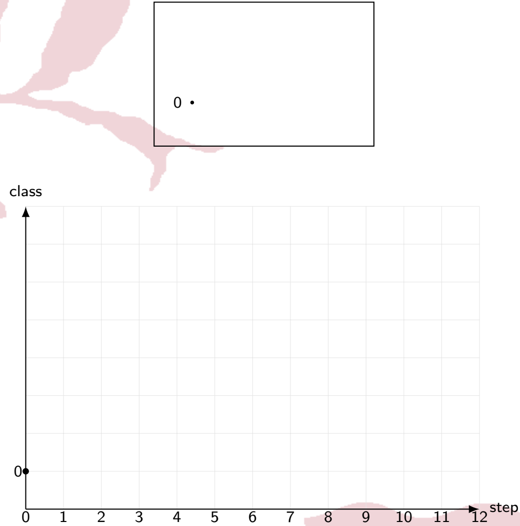

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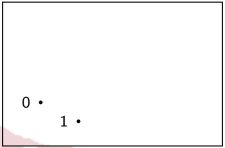

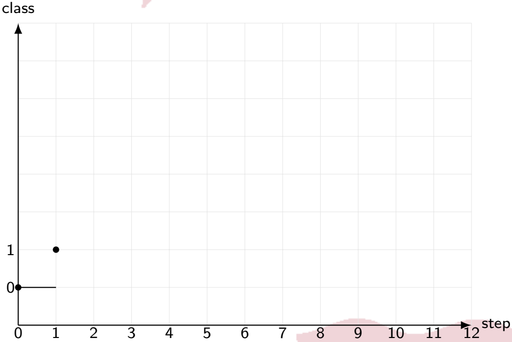

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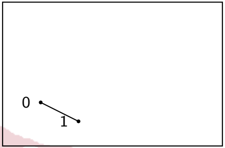

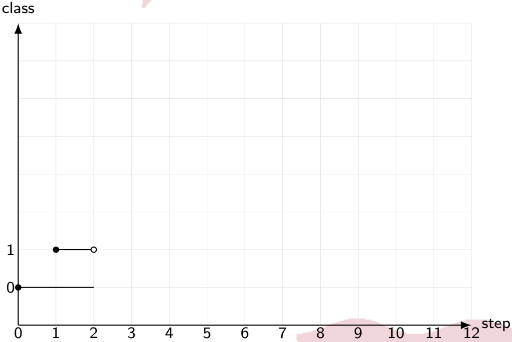

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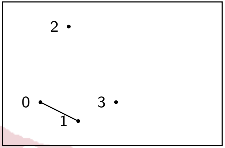

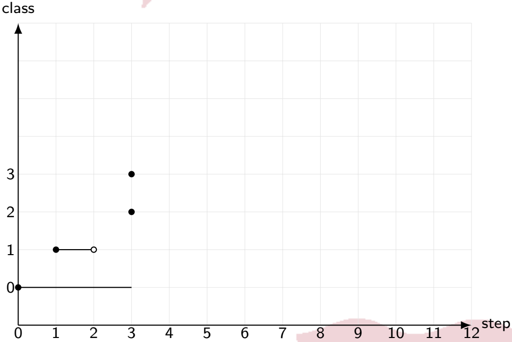

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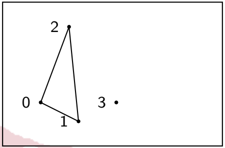
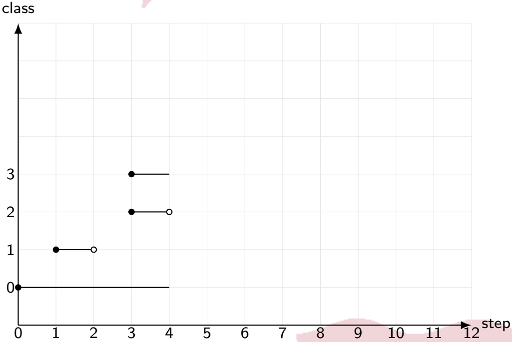
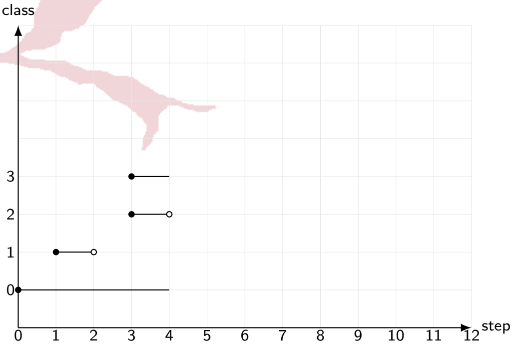




Example

. . .



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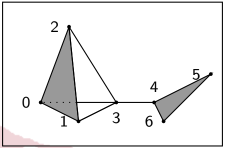
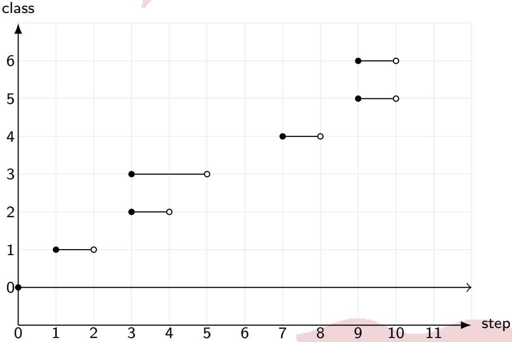
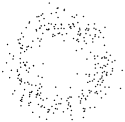




Application: Point-Cloud datasets



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Construct a filtration from the point-cloud dataset :

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- Vietoris-Rips Complexes
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- Alpha Complexes
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G. Singh et. al. *Topological analysis of population activity in visual cortex.* Journal of Vision 8:8(2008).
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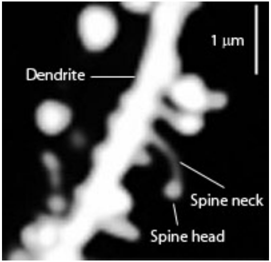
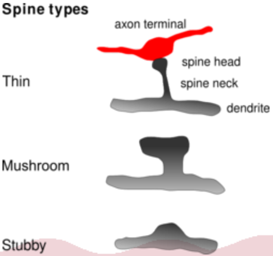
- The dendrites of a single neuron can contain hundreds to thousands of spines





The problem

Goal:

- Automatic measurement and classification of spines of a neuron

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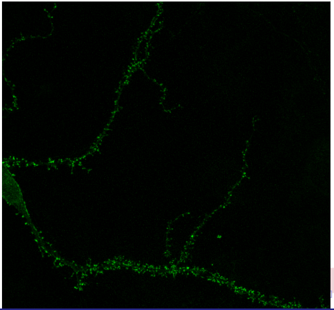
First step:

- Detect the region of interest (the dendrites)
- Main problem: noise
  - salt-and-pepper
  - non-relevant biological elements



Outline

Getting the images

Optical sectioning:

- Produces clear images of a focal planes deep within a thick sample
- Reduces the need for thin sectioning
- Allows the three dimensional reconstruction of a sample from images captured at different focal planes

- `http://loci.wisc.edu/files/loci/8OpticalSectionB.swf`

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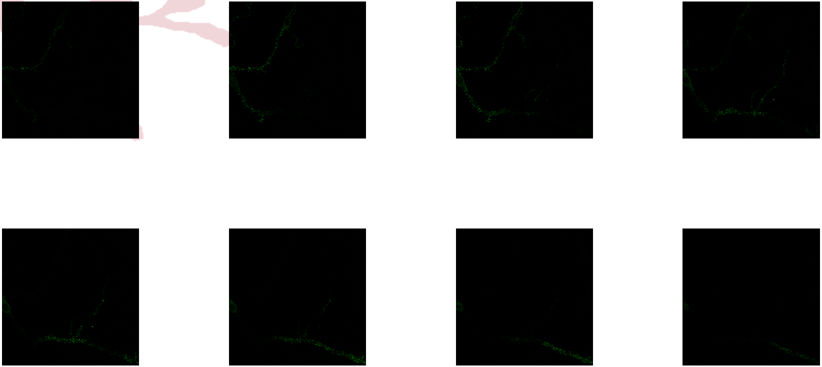
Procedure:

1. Get a stack of images using optical sectioning
2. Obtain its maximum intensity projection

Example

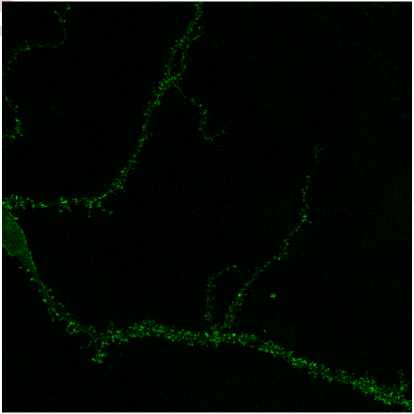
Stack of images:



Important feature: the neuron persists in all the slices

Example

Z-projection:



Our method

Our method:

1. Reduce salt-and-pepper noise
2. Dismiss irrelevant elements as astrocytes, dendrites of other neurons, and so on

Our method: Reducing salt-and-pepper noise

Salt-and-pepper noise:

- Produced when captured the image from the microscope

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Salt-and-pepper noise:

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- Solution:
  1. Low-pass filter:
     - Decrease the disparity between pixel values by averaging nearby pixels
     - uniform, Gaussian, median, maximum, minimum, mean, and so on
     - In our case (after experimentation): median filter with a radius of 10 pixels

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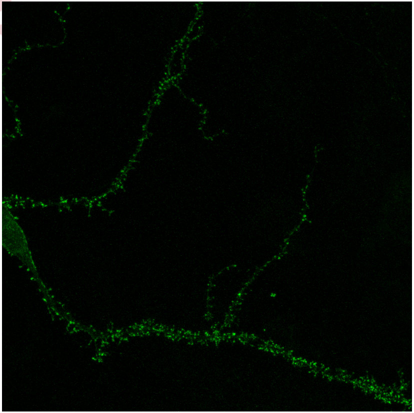
     - In our case (after experimentation): median filter with a radius of 10 pixels
  2. Threshold:
     - Discrimination of pixels depending on their intensity
     - A binary image is obtained
     - In our case (after experimentation): Huang's method

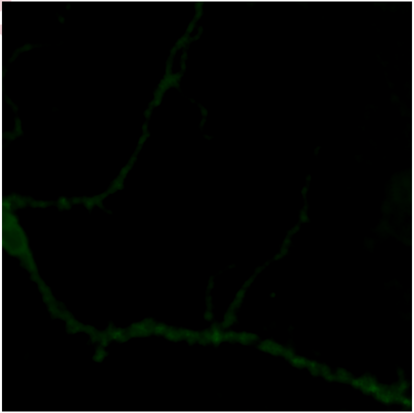
Our method: Reducing salt-and-pepper noise

Original image:



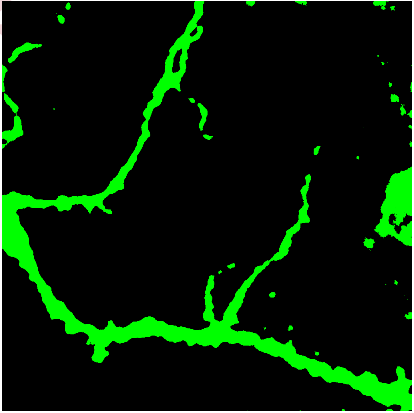
Our method: Reducing salt-and-pepper noise

After low-pass filter:



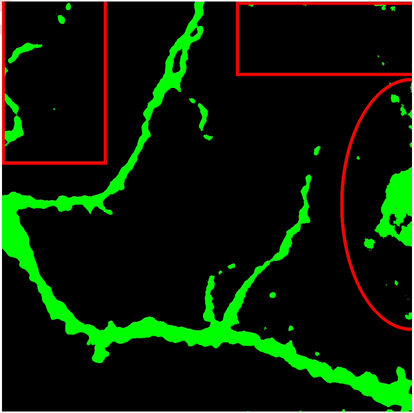
Our method: Reducing salt-and-pepper noise

After threshold:



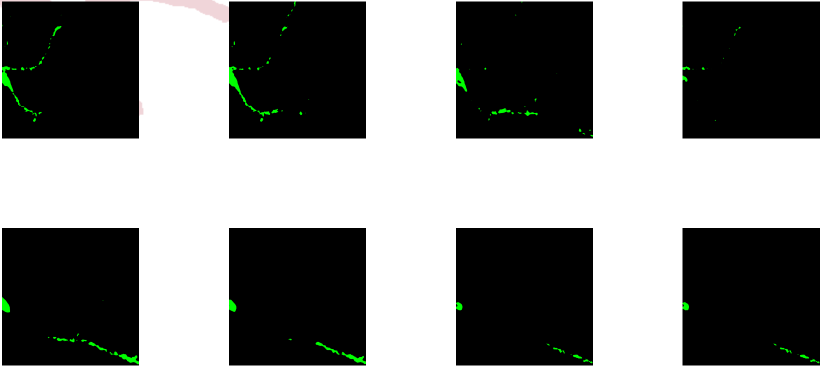
Our method: Reducing salt-and-pepper noise

Undesirable elements:

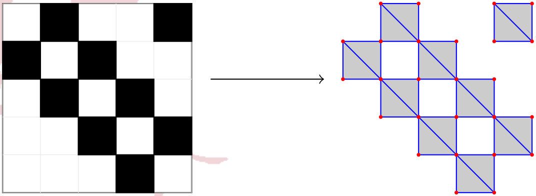


Our method: Reducing salt-and-pepper noise

Preprocessing is applied to all the slices of the stack:



Simplicial Complexes from Digital Images



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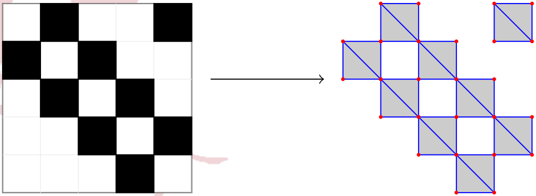

A monochromatic image $\mathcal{D}$:

- set of black pixels
- a *subimage* of $\mathcal{D}$ is a subset $\mathcal{L} \subseteq \mathcal{D}$
- a *filtration of an image* $\mathcal{D}$ is a nested subsequence of images

$$D^0 \subseteq D^1 \subseteq \ldots \subseteq D^m = \mathcal{D}$$

- a filtration of an image induces a filtration of simplicial complexes

A filtration of the Z-projection

Let us construct a filtration from the processed Z-projection:

A filtration of the Z-projection

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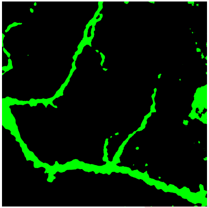
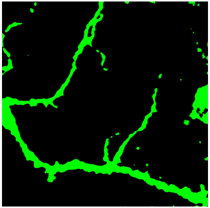
1. $\mathcal{D} = D^m$ is the processed Z-projection

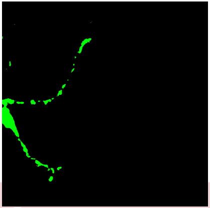


A filtration of the Z-projection

Let us construct a filtration from the processed Z-projection:

1. $\mathcal{D} = D^m$ is the processed Z-projection
2. D^{m-1} consists of the connected components of D^m such that its intersection with the first slide is not empty





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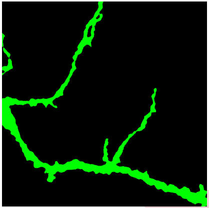

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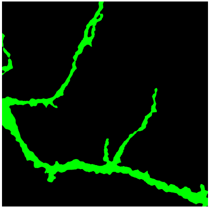

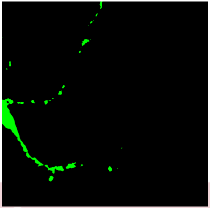

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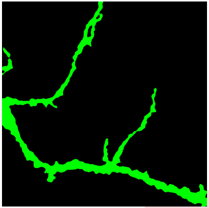

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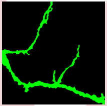
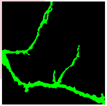
4. ...

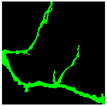
A filtration of the Z-projection



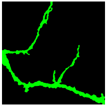
D^0 $\subseteq$



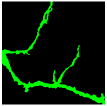
D^1 $\subseteq$



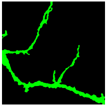
D^2 $\subseteq$



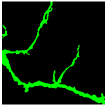
D^3 $\subseteq$



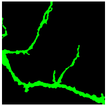
D^4 $\subseteq$



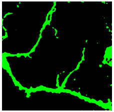
D^5 $\subseteq$



D^6 $\subseteq$



D^7 $\subseteq$



D^8

A filtration of the Z-projection

Remember: the neuron persists in all the slices

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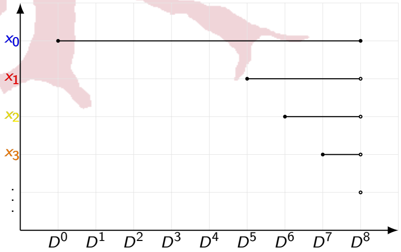
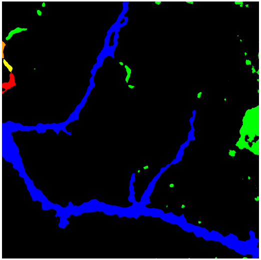
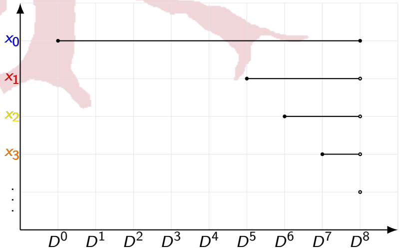
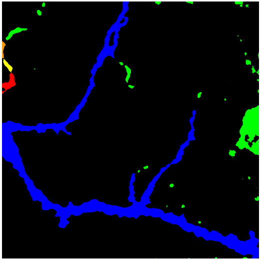




A filtration of the Z-projection

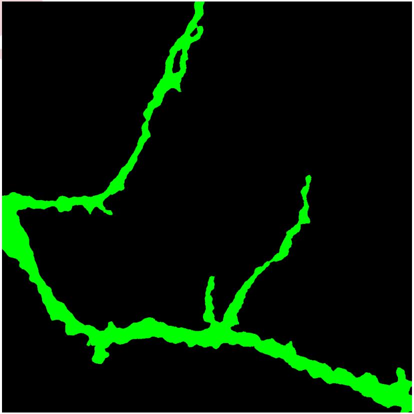
Remember: the neuron persists in all the slices



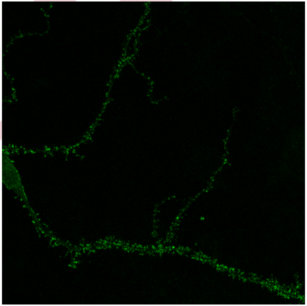


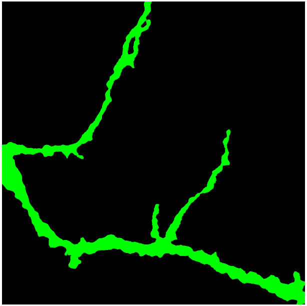
The components of the neuron persist all the life of the filtration

Final result



Final result





Outline

Conclusions and Further work

Conclusions:

- Method to detect neuronal structure based on persistent homology

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- Method to detect neuronal structure based on persistent homology

Further work:

- Implementation of an ImageJ plug-in
- Intensive testing
- Software verification?
- Measurement and classification of spines
- Persistent Homology $\leftrightarrow$ Discrete Morse Theory

Neuronal structure detection using Persistent Homology*

J. Heras, G. Mata and J. Rubio

Department of Mathematics and Computer Science, University of La Rioja

Seminario de Informática Mirian Andrés
March 20, 2012

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