Statistical Machine Learning in Interactive Theorem Proving

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(Funded by EPSRC First Grant Scheme)

University of Dundee

18 July 2014
1. Existing tools and challenges
Outline

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2. ML4PG: “Machine Learning for Proof General”
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2. ML4PG: “Machine Learning for Proof General”
3. Conclusions and Further work
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2. ML4PG: “Machine Learning for Proof General”
3. Conclusions and Further work
4. Using ML4PG: Demo
Challenges for ITP users:

- . . . size and sophistication of libraries stand on the way of efficient knowledge reuse;
  - . . . applied in formal mathematical proofs: Four Colour Theorem (60,000 lines), Kepler conjecture (325,000 lines), Feit-Thompson Theorem (170,000 lines), etc.
  - . . . applied in industrial proofs: seL4 microkernel (200,000 lines), verified C compiler (50,000 lines), ARM microprocessor (20,000 lines), etc.

- . . . manual handling of various proofs, strategies, libraries, becomes difficult;
- . . . team-development is hard, especially as ITPs are sensitive to notation;
- . . . comparison of proof similarities is hard.
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- ... comparison of proof similarities is hard.
Existing tools and challenges

Existing tools for managing Coq libraries: Search

Several searching tools in Coq:
- **Search**, SearchAbout, SearchPattern and SearchRewrite.
- SSReflect implements its own version **Search** with functionality the 4 Coq’s search commands.

Example

```
Search "distr" in bigop
Search _ (_ * (\big[_/__]_(_ <- _| _)_))
```

- The Whelp platform is a web search engine in Coq code, with 3 functions:
  - Match (similar **Search**),
  - Hint (finds all the theorems which can be applied to derive the current goal) and
  - Elim (retrieves all the eliminators of a given type).
Main properties of the search engines:

- goal oriented
- hence the user should already know what he is searching for: pattern/library/lemma name, etc
- deterministic: if the exact requested pattern exists, they will find it.
Existing tools and challenges

Existing tools-2: Dependency graphs

- You do not have to know what you are searching for
- They show “all there is”.
Existing tools-2: Dependency graphs

or perhaps all there is relative to your lemma/term
Existing tools and challenges

Dependency graphs DOs and DON’Ts

- nicely visualised
- not goal directed – but can be used for a goal
- deterministic: if there is a dependency, it will be shown
- but it would not tell you if there are similar lemmas/terms
- it would not tell you which of those dependencies are more important than others for the proof
- there may be excessive information that actually hides the essence of the proof
The missing tool...

Something that could help us to:

- capture and search the *meaning* of the libraries;
- the higher-level proof strategies beyond tactics and notations
- identify redundancies and repetitions....
Motivating example:

**Theorem (Fundamental Lemma of Persistent Homology)**

\[ \beta_{i,k}^{j} : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{Z} \]

\[ \beta_{n}^{k,l} - \beta_{n}^{k,m} = \sum_{1 \leq i \leq k} \sum_{l < j \leq m} (\beta_{n}^{j,p-1} - \beta_{n}^{j,p}) - (\beta_{n}^{j-1,p-1} - \beta_{n}^{j-1,p}) \]

Apply case on \( n \).

1. **Prove the base case (a simple task).**
2. **Prove the case** \( 0 < n \):
   1. expand the summation,
   2. cancel the terms pairwise,
   3. the only terms remaining after the cancellation are the first and the last one.
Same strategy:

**Lemma**

Let $M$ be a nilpotent matrix, then

$$(1 - M) \times \sum_{0 \leq i < n} M^i = 1$$

where $n$ is such that $M^n = 0$

**Lemma**

If $g : \mathbb{N} \to \mathbb{Z}$, then

$$\sum_{0 \leq i \leq k} (g(i + 1) - g(i)) = g(k + 1) - g(0)$$

**Lemma**

Let $M$ be a nilpotent matrix, then there exists $N$ such that $N \times (1 - M) = 1$
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4. Using ML4PG: Demo
Proof pattern recognition in ITPs

Goal: make machine-learning a part of interactive proof development

Apply machine-learning methods to:

- find common proof-patterns in proofs across various scripts, libraries, users and notations;
- provide proof-hints in real-time and relative to a proof stage;
- assist the user, not the prover.
Proof pattern recognition in ITPs

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ML4PG:

- Proof General extension which applies machine learning methods to Coq/SSReflect proofs. [Now available in standard Proof General distribution]

Machine Learning 4 Proof General: interfacing interfaces

Proof General

ML4PG

Interactive Prover: Coq, SSReflect

proof families

MATLAB/Weka

Clustering: K-means, Gaussian, ...

feature extraction

F.1. works on the background of Proof General extracting some low-level features from proofs in Coq/SSReflect.

F.2. automatically sends the gathered statistics to a chosen machine-learning interface and triggers execution of a clustering algorithm of user’s choice;

F.3. does some post-processing of the results and displays families of related proofs to the user.
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ML4PG approach to proof-clustering

We have integrated Proof General with a variety of clustering algorithms:
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We have integrated Proof General with a variety of clustering algorithms:

- Unsupervised machine learning technique:

- Engines: Matlab, Weka, Octave, R, ...
- Algorithms: K-means, Gaussian Mixture models, simple Expectation Maximisation, ...
Feature extraction and output strategies

General pattern-search

Coq libraries: one or many?

Terms → Visualise term clusters?

Proofs → Visualise proof families?

Terms → Visualise the term tree?

Proofs → Visualise the proof flow?

Relative to Coq object
Feature extraction

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Feature Extraction for Terms

- specifically developed for Coq and ML4PG

Terms are understood broadly: ...Definitions, type declarations, (co)fixpoint function definitions, lemma and theorem statements...

Example

\[ \forall (n : \text{nat}) (H : \text{even } n), \text{odd } (n + 1). \]
Feature Extraction for Terms

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Terms are understood broadly: ...Definitions, type declarations, (co)fixpoint function definitions, lemma and theorem statements...

Example

\[
\text{forall } (n : \text{nat}) (H : \text{even } n), \text{odd } (n + 1).
\]

Convert to a term-tree:

```
forall

| n : nat

| H : even n

| odd : nat -> Prop

| + : nat -> nat -> nat

| n : nat

| 1 : nat
```
What are the important features of a term tree?

\[
\forall n : \text{nat} \quad H : \text{even } n \quad \text{odd} : \text{nat} \rightarrow \text{Prop}
\]

\[
+ : \text{nat} \rightarrow \text{nat} \rightarrow \text{nat}
\]

\[
n : \text{nat} \quad 1 : \text{nat}
\]

its topology
What are the important features of a term tree?

1. its topology
2. what populates its nodes:
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   - definitions of those terms and types, their **meaning** and structure...
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2. what populates its nodes:
   - terms and their types
   - definitions of those terms and types, their meaning and structure...
   - their role in this proof library...
### The method of Recurrent Clustering:

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  - ...starts with Gallina pre-defined symbols, and uses them to find similarity of the first few Coq definitions; and then proceeds recursively.
- The more two terms or types are "**semantically similar**", the closer values they get. Thus, this matrix should have similar content to e.g. the matrix of forall (n : nat) (H : odd n), even (n + 1).
Motivation for this feature extraction:

For each term, we get a matrix of size up to 300 features, which capture:

1. term-tree structure of that term – via the term-depth, level-size, and additional “third” feature relating the above;
2. its types as related to terms; (pattern-recognition tools analyse the relative values of all features, as 1 Coq object is a point in 300-dimensional space)
3. its dependency to other definitions and Coq terms types – via recurrent clustering
A simple example

General library clustering:

SSReflect Base library, 12 standard files, 457 terms, 91 clusters (the number and size of clusters can be changed using PG options); 5-10 seconds.

```
Fixpoint eqn (m n : nat) :=
match m, n with
| 0, 0 => true
| m'.+1, n'.+1 => eqn m' n'
| _, _ => false end.

Fixpoint eqseq (s1 s2 : seq T) :=
match s1, s2 with
| [:], [:] => true
| x1 :: s1', x2 :: s2' => (x1 == x2) && eqseq s1' s2'
| _, _ => false end.
```

Note: common structure across types and type constructors
A simple example

General library clustering:

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Example

Fixpoint drop n s := match s, n with
| _ :: s’, n’.+1 => drop n’ s’
| _, _ => s end.

Fixpoint take n s := match s, n with
| x :: s’, n’.+1 => x :: take n’ s’
| _, _ => [::] end.

Intuitive...
A simple example

General library clustering:

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Example

Definition flatten := foldr cat (Nil T).
Definition sumn := foldr addn 0.

Analyses deep into structures of subterms by recurrent clustering: cat and addn are defined on lists and natural numbers, but are in the same cluster recurrently. These 2 grouped together out of 15 other definitions using foldr.

- Goal-oriented clustering: do the same but show only what is related to certain Coq object: e.g. related to flatten. (See demo)
ML4PG: “Machine Learning for Proof General”

Coq libraries: one or many?

General pattern-search

Terms → Visualise term clusters?

Proofs → Visualise proof families?

Terms → Visualise the term tree?

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Relative to Coq object
Proof-clustering

Similarly to term-clustering, the feature extraction:

- relies on recurrently computed features;
- considers a fragment of a proof-tree – a proof-patch – to find relative dependencies between goals, tactics and tactic arguments;
- considers relation of the tactic arguments to the (inductive) hypotheses or library lemmas.

As before,

- a 5-step proof patch is given by 85 features;
- each proof patch is a point in 85-dimensional space;
- the proof pattern is determined by looking at their correlation in several proofs.
A proof-feature algorithm by example

HoTT Path library

Lemma dpath_path_l A : Type x1 x2 y : A
  (p : x1 = x2) (q : x1 = y) (r : x2 = y) :
  q = p @ r `<~>` transport (fun x => x = y) p q = r.

Proof.
  destruct p; simpl.
  exact (equiv_concat_r (concat_1p r) q).
Qed.

Lemma transport_paths_lr A : Type x1 x2 : A (p : x1 = x2) (q : x1 = x1) :
  transport (fun x => x = x) p q = p^ @ q @ p.

Proof.
  destruct p; simpl.
  exact (((concat_1p q)^ @ (concat_p1 (1 @ q))^ ).
Qed.
A proof-feature algorithm by example

HoTT Path library

Lemma dpath_path_1 A : Type x1 x2 y : A
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<th>arg</th>
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<td>([&lt;~&gt;] term, [=] term, [=] term)</td>
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Proof-patch analysis

Knowing how to prove Lemma \texttt{dpath\_path\_1}, what else can I prove?
ML4PG: visualisation

Coq libraries: one or many?

General pattern-search

Terms

Visualise term clusters?

Proofs

Visualise proof families?

Terms

Visualise the term tree?

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Visualisation: HoTT library Term Similarity Graph
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- Request a proof-hint during an on-going proof-development;
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- Find how different proof strategies in two libraries about the same subject are;
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- Detect proof-patterns prior to new library development
- Request a proof-hint during an on-going proof-development;
- Import a proof methodology from a different library;
- Find how different proof strategies in two libraries about the same subject are;
- Share work-load in a team development.


Benefits of this approach:

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- modular: allows the user to make choices regarding approach to levels of proofs and particular statistical algorithms;
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- does not assume any knowledge of machine-learning interfaces from the user;
- modular: allows the user to make choices regarding approach to levels of proofs and particular statistical algorithms;
- tolerant to mixing and matching different proof libraries and different notation used in proofs across different users.
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- modular: allows the user to make choices regarding approach to levels of proofs and particular statistical algorithms;
- tolerant to mixing and matching different proof libraries and different notation used in proofs across different users.
- ML4PG is now a part of standard Proof General distribution.
### ML4PG in comparison

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## Conclusions and Further work

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### Conclusions and Further work

#### Percentage of atomically re-proven theorems:

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<th>Granularity 3</th>
<th>Granularity 5</th>
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<tr>
<td>ssrnat (SSReflect)</td>
<td>48%</td>
<td>36%</td>
<td>28%</td>
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<tr>
<td>seq (SSReflect)</td>
<td>21%</td>
<td>20%</td>
<td>15%</td>
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<tr>
<td>ssrbool (SSReflect)</td>
<td>70%</td>
<td>77%</td>
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<td>fintype (SSReflect)</td>
<td>7%</td>
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<tr>
<td>JVM</td>
<td>56%</td>
<td>58%</td>
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<tr>
<td>summations</td>
<td>0%</td>
<td>10%</td>
<td>12%</td>
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<tr>
<td>Paths (HoTT)</td>
<td>92%</td>
<td>91%</td>
<td>94%</td>
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<tr>
<td>Nash Equilibrium</td>
<td>40%</td>
<td>37%</td>
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Outline

1. Existing tools and challenges
2. ML4PG: “Machine Learning for Proof General”
3. Conclusions and Further work
4. Using ML4PG: Demo
Statistical Machine Learning in Interactive Theorem Proving

Katya Komendantskaya and Jonathan Heras
(Funded by EPSRC First Grant Scheme)

University of Dundee

18 July 2014
Can we use ML4PG to automatically prove theorems?

Given the statement of a theorem $T$ and the associated library $L$, we can use ML4PG to try to find a proof for $T$ as follows:

1. Use ML4PG to obtain the cluster $C$ from the library $L$ that contains the theorem $T$.
2. Obtain the sequence of tactics $\{ T^i_1, \ldots, T^i_{n_i} \}_{i}$ used to prove each lemma in $C$.
3. For each $i$, try to prove $T$ using $T^i_1, \ldots, T^i_{n_i}$.
4. If no sequence of tactics prove $T$, then for each tactic use ML4PG to infer the argument for each tactic $T^i_j$:
   - If the argument of $T^i_j$ is an internal hypothesis from the context of a proof, try all the internal hypothesis from the context of the current proof.
   - If the argument of $T^i_j$ is an external lemma $L$, use ML4PG to compute all the lemmas in the same cluster as $L$ and try all those lemmas.

*** This can be naturally extended to tactics with several arguments, just trying all the possible combinations.